

Development of physically informed neural network (PINN) interatomic potentials

Speaker: J. Hickman*

G. Purja Pun, F. Tavazza*, Y. Mishin****

* National Institute of Standards and Technology (NIST)
Materials measurement laboratory (MML)

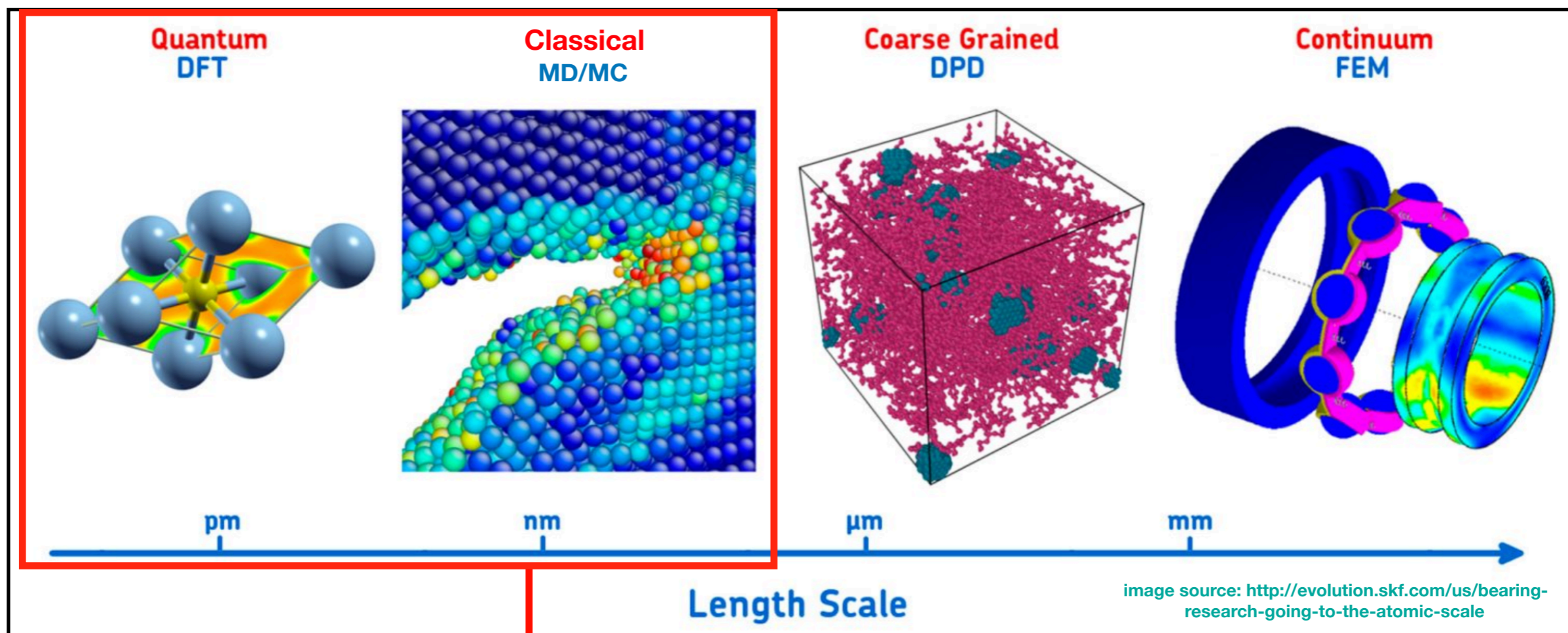
** George Mason University: Department of Physics

LAMMPS Workshop and Symposium

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Supported by NRC postdoctoral
fellowship

Motivation



Ab-initio/DFT

- Slow
- Size limited ($N \sim 10^2$)
- 0K or short timescales
- Very accurate

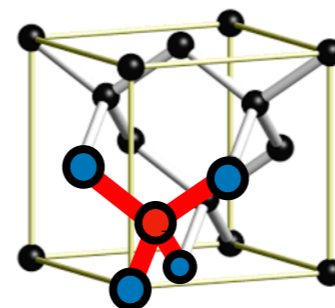
Compromises

Speed
Accuracy
Scalability
Transferability

Better potential models

Classical (MD/MC)

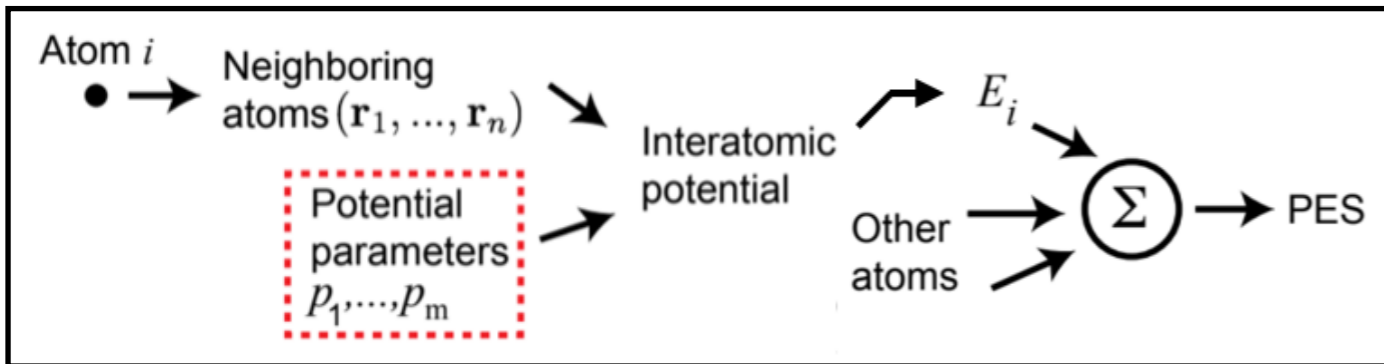
- Computationally Fast
- Larger systems ($N \sim 10^7$)
- Kinetic phenomena/long simulations
- ★ Accuracy depends on *approximation* of the potential energy surface (PES)



$$E = E(\mathbf{r}_1, \dots, \mathbf{r}_N)$$

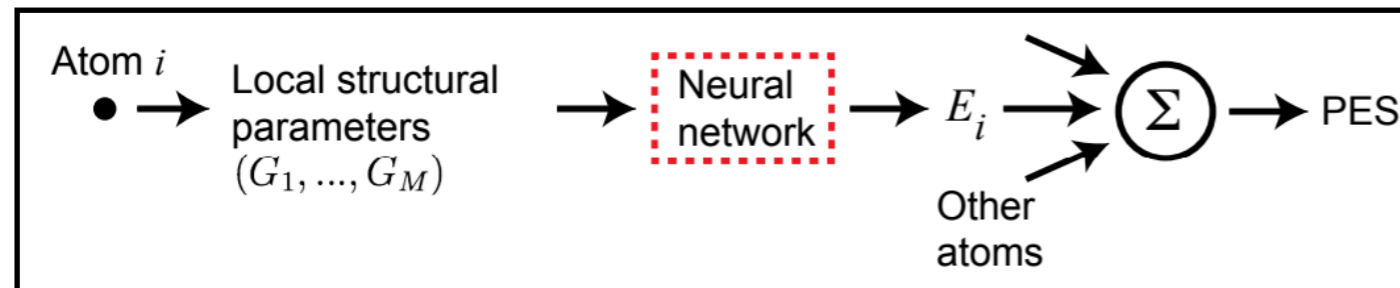
Interatomic potential model types

Traditional interatomic potential:



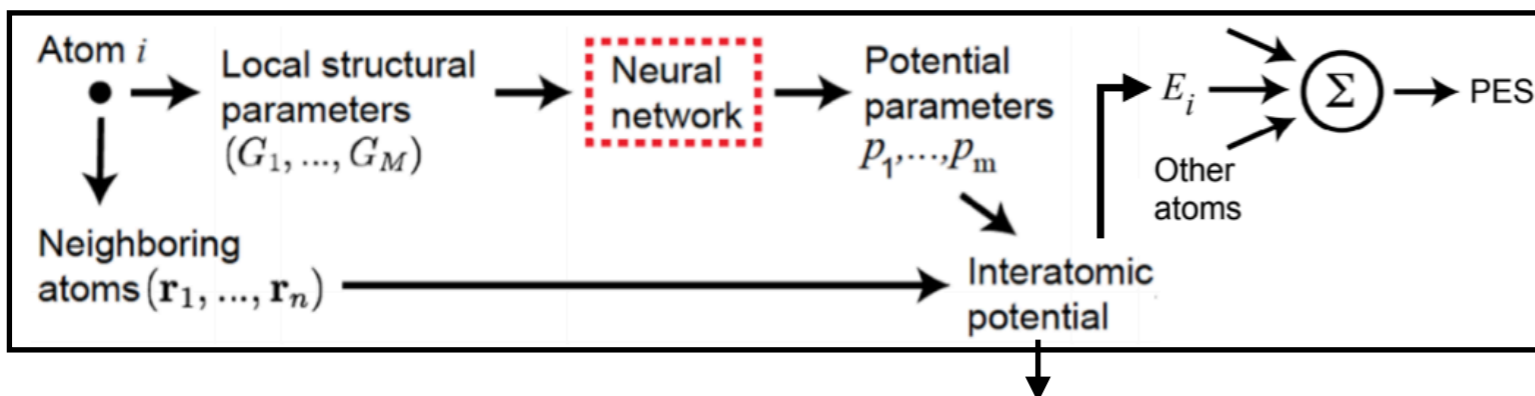
- LJ, EAM, ADP, Tersoff, REAX ... etc

“Mathematical” or “straight” NN potentials:



- Machine learning potentials
 - Gaussian process regression
 - Interpolating moving least squares
 - Kernel ridge regression
 - Compressed sensing
 - ANN potentials

Physically informed neural network (PINN):



local parameterization

Pros

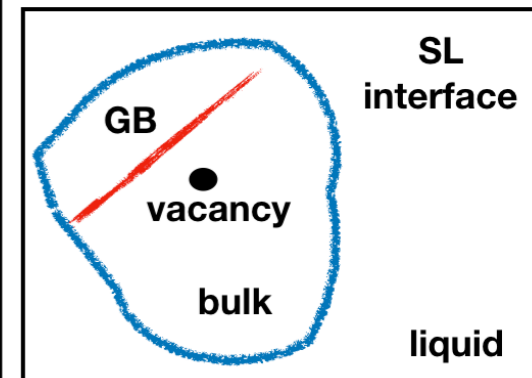
- Fast
- Decent extrapolation
- Physically inspired

- Fast relative to DFT
- DFT level accuracy (~1-5 mEv) within training set
- Relatively straight forward/routine to train/fit
- Systematic improvement (add more data)

- Same as straight NN
- Decent extrapolation
- Physically inspired

Cons

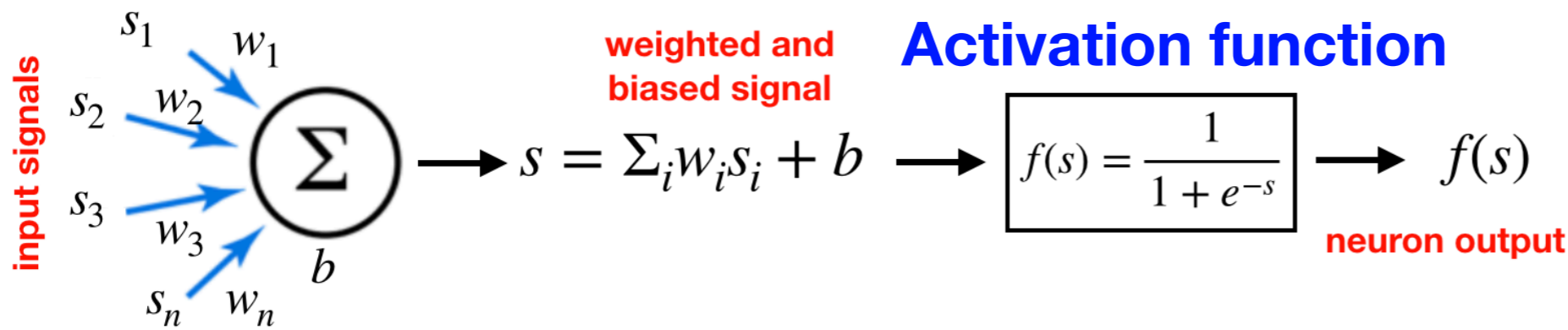
- Difficult to train/fit
- Hard to improve upon once finalized
- Accuracy limitations
- Slower than traditional potentials
- Bad extrapolation



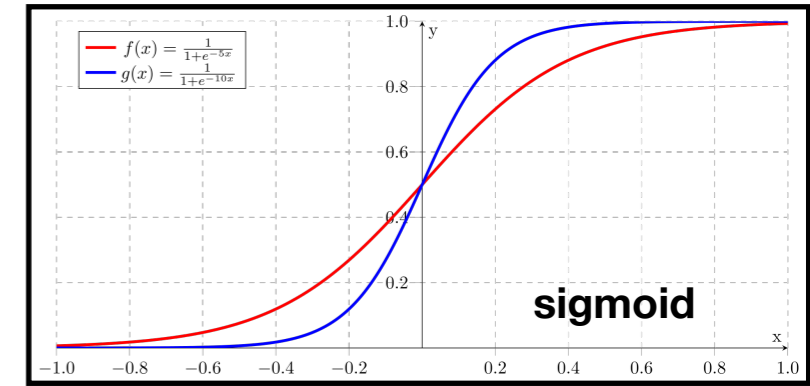
- Slower than traditional potentials

Artificial neural networks

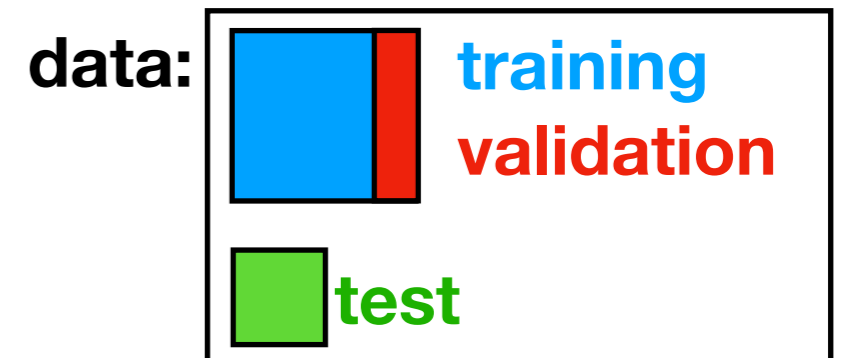
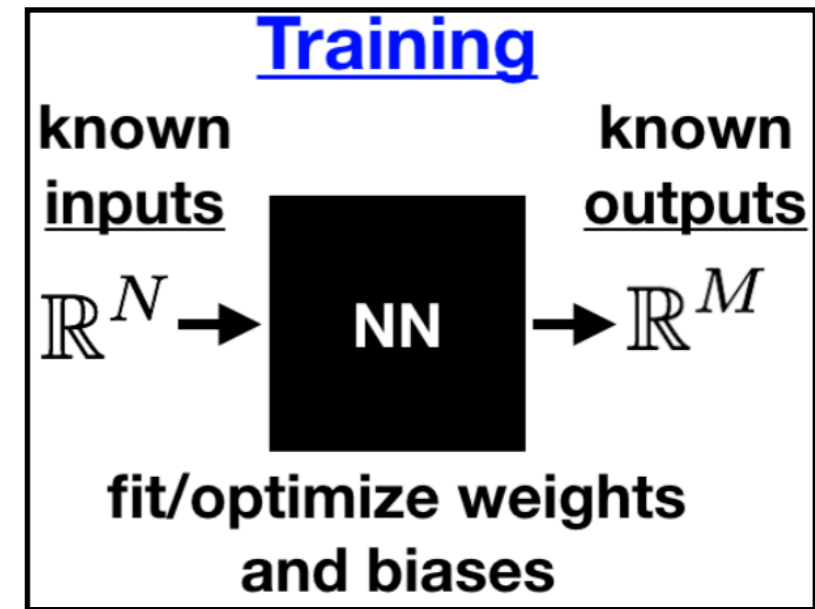
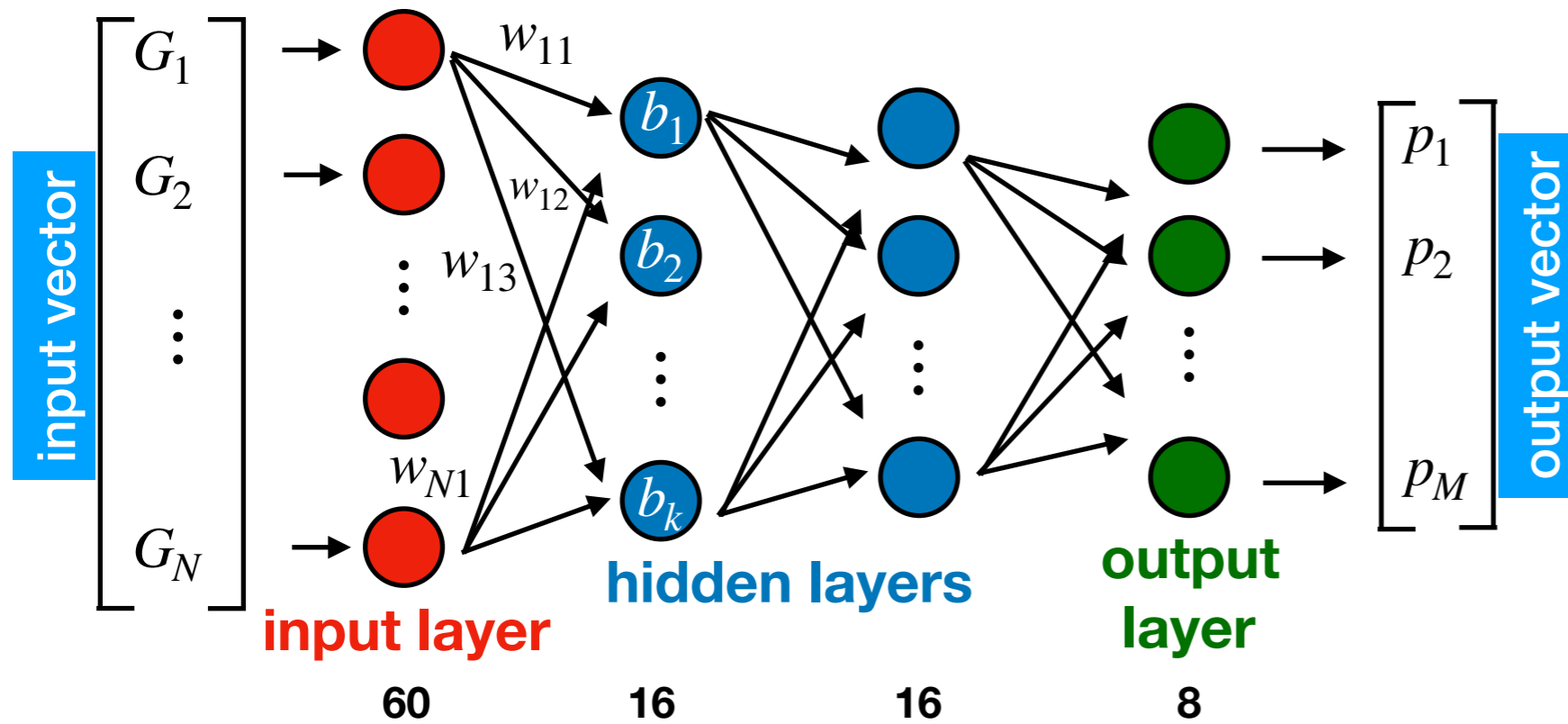
Artificial neuron AKA: nodes or perceptrons



Activation function



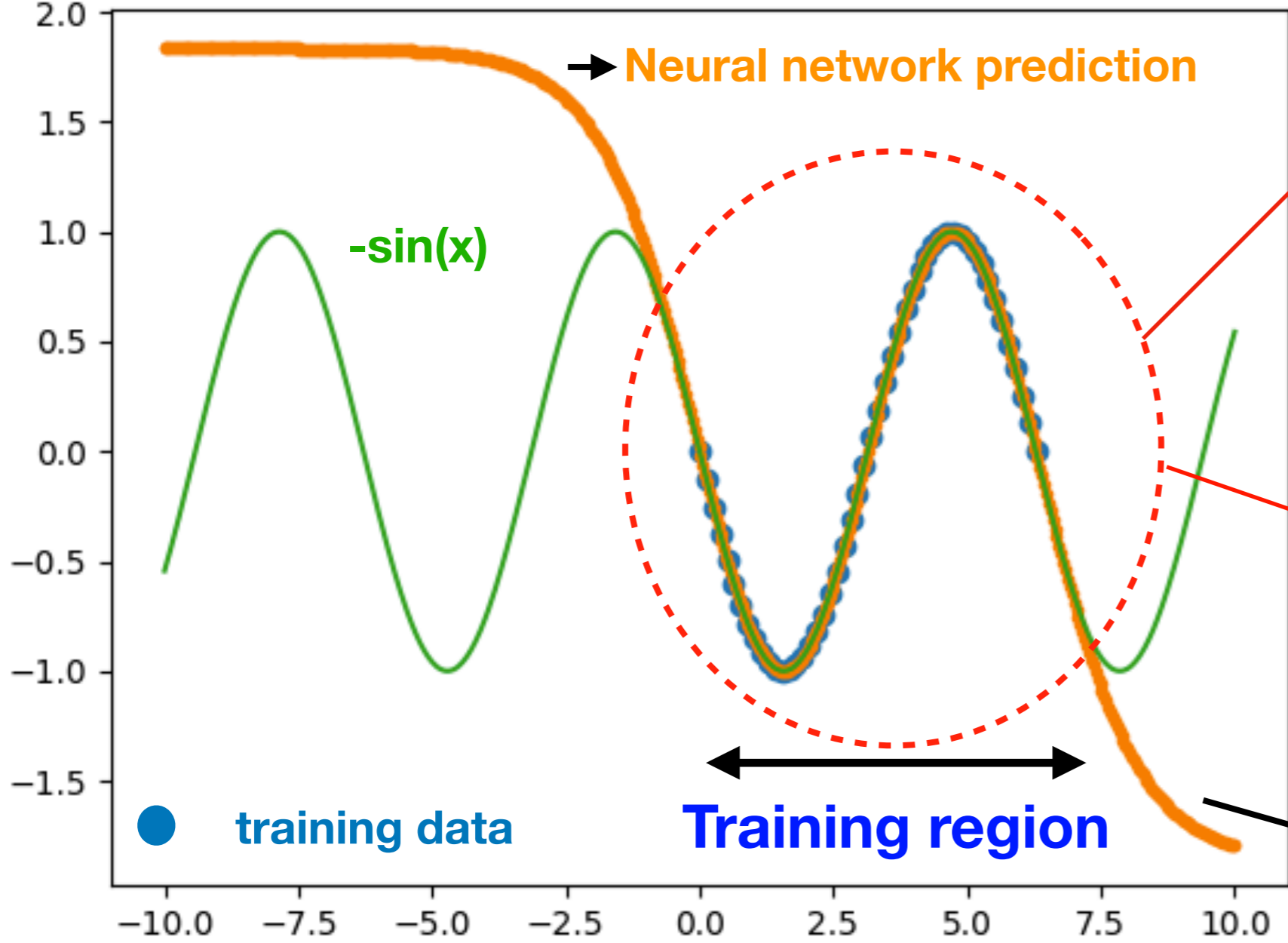
Artificial neural network



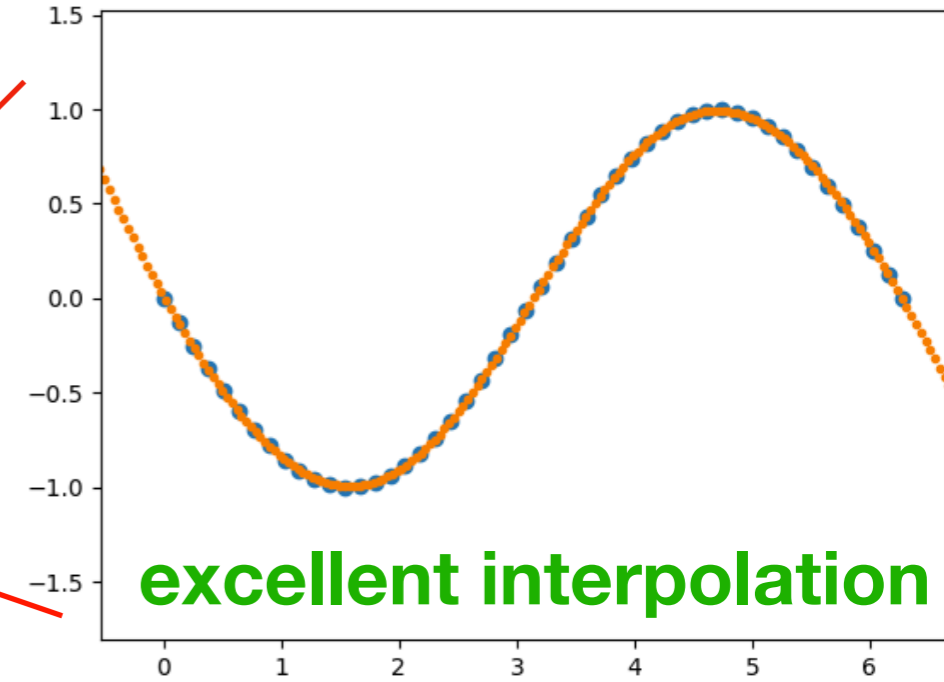
The weights and bias are the NN's fitting parameters (~1500 parameters)

Artificial neural networks

Toy example:



Train NN to reproduce provided data



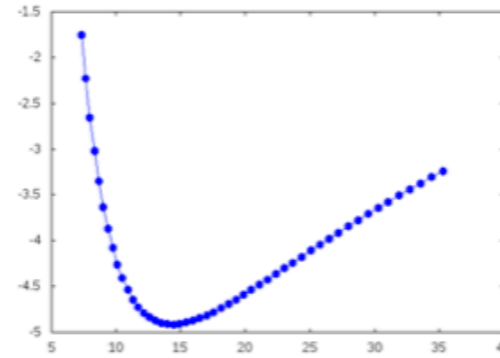
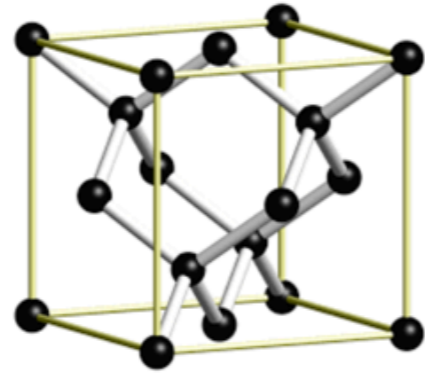
unable to extrapolate



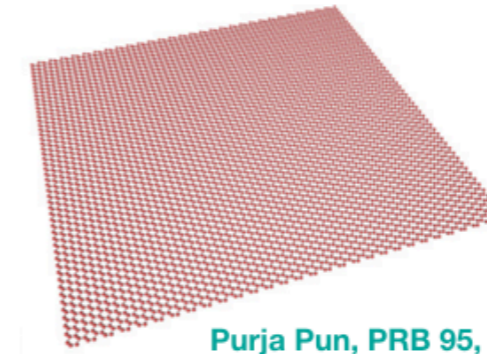
Physically informed neural network potential (PINN)

Training/test set generation

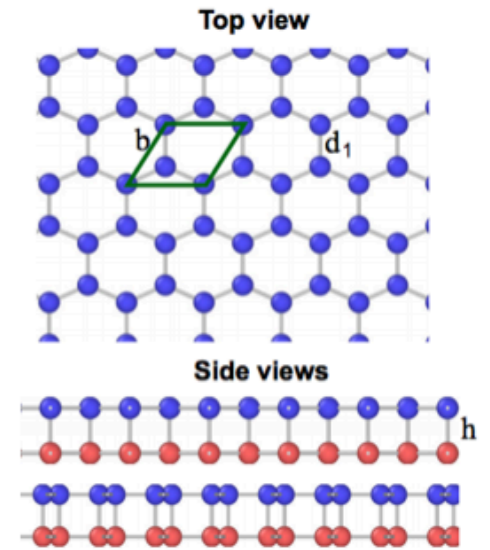
Stable structure: (Diamond)



Two dimensional structures:

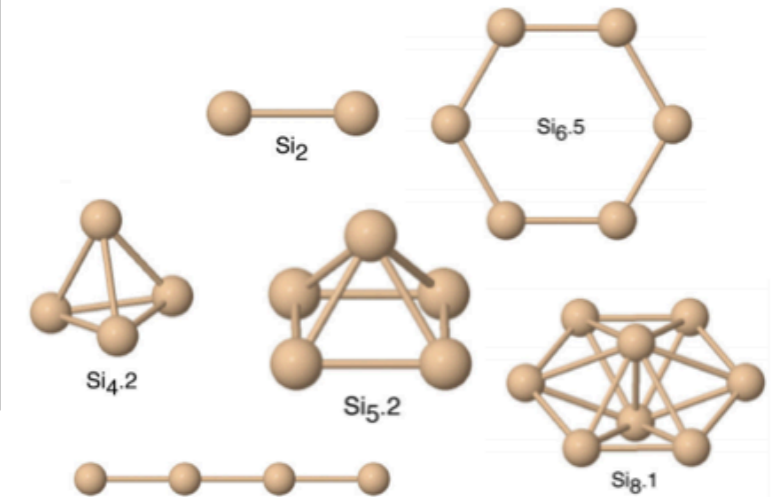


Purja Pun, PRB 95, 224103 (2017)



6 silicene allotropes

Clusters:



Purja Pun, PRB 95, 224103 (2017)

~18 atomic clusters

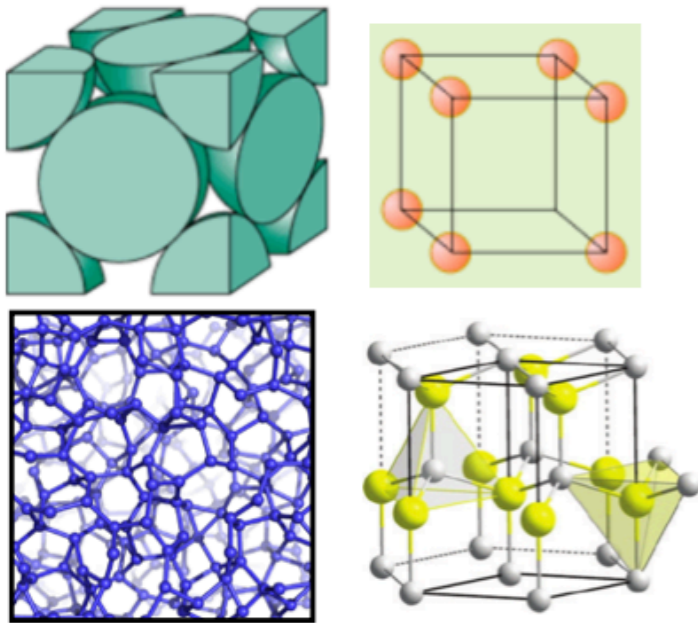
DFT calculation details

- Functional/PP: PBE/PAW
- ENCUT=600 eV
- ~4300 structures
- block size 1-96 atoms
- k-point convergence tests for each group

Non-equilibrium sampling

- Isotropic expansions/compressions
- Random local atomic perturbations
- Anisotropic box variations

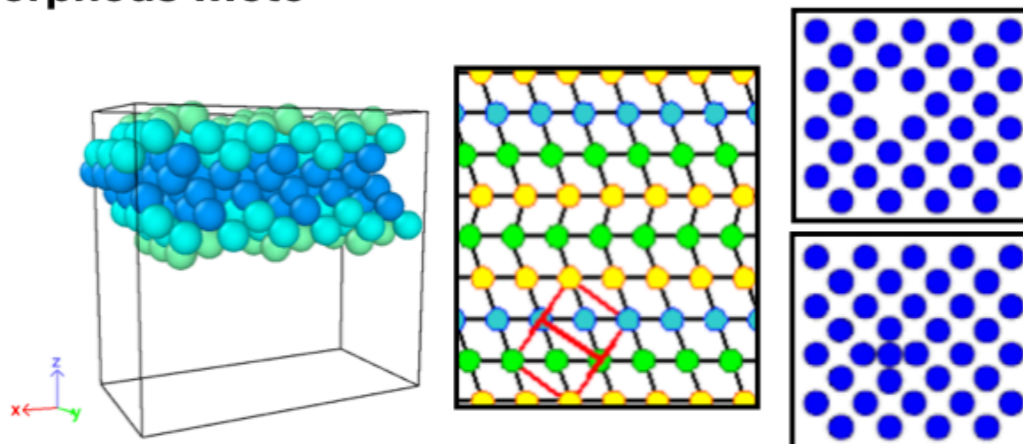
Alternative structures:



~14 alternate structures

- FCC, BCC, HEX, HCP, SC, Liquid, Amorphous ...etc

Defects:

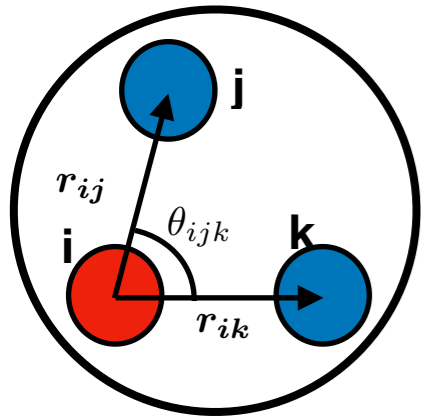


~27 different defects

- Vacancies, Various self interstitials, Surfaces, Stacking faults

- DFT data is shifted so DC phase coincides with -4.63

Traditional BOP potential



$$E_i = \frac{1}{2} \left(\sum_{j \neq i} e^{A_o - \alpha_o r_{ij}} - S_{ij} b_{ij} e^{B_o - \beta_o r_{ij}} \right) f_c(r_{ij}) + E_p$$

cutoff function \uparrow

bond order parameter $b_{ij} = (1 + z_{ij})^{-\frac{1}{2}}$

promotional energy \downarrow

$$E_p = -\sigma_o \left(\sum_{j \neq i} S_{ij} b_{ij} f_c(r_{ij}) \right)^{\frac{1}{2}}$$

$$z_{ij} = a_o \sum_{k \neq i, j} S_{ik} (\cos(\theta_{ijk}) + h_o)^2 f_c(r_{ik})$$

$$S_{ijk} = 1 - f_c(r_{ik} + r_{jk} - r_{ij}) e^{-\lambda_o (r_{ik} + r_{jk} - r_{ij})} \quad \text{screening} \quad S_{ij} = \prod_{k \neq i, j} S_{ijk}$$

step-0:

Fit a baseline traditional potential via 8 adjustable $(A_o, B_o, \alpha_o, \beta_o, a_o, h_o, \lambda_o, \sigma_o)$

Training process:

pytorch

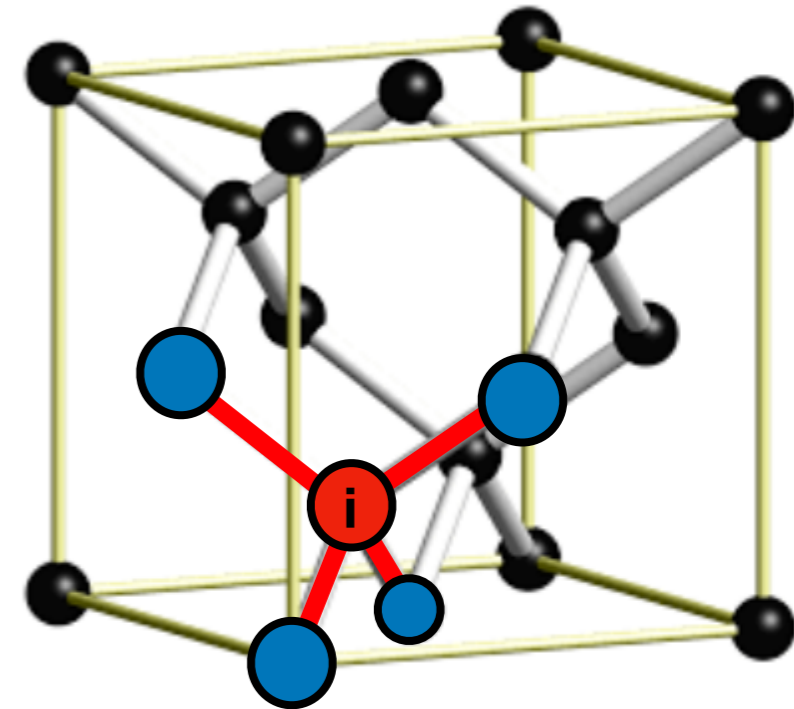
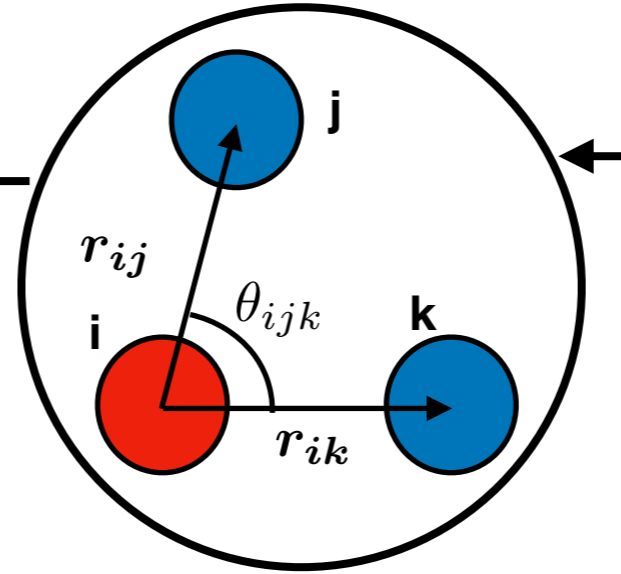
	"Training set"		Model error
	Structures	DFT energies	Model energies
structure-1		E_1	\tilde{E}_1
structure-2		E_2	\tilde{E}_2
⋮		⋮	⋮
structure-N		E_N	\tilde{E}_N

$$RMSE = \left(\frac{\sum_s (E_s - \tilde{E}_s)^2}{N} \right)^{\frac{1}{2}}$$

Local structure parameters

Structure parameters:

$$G_i^{m,n} = \sum_{j,k} P_m(\cos(\theta_{ijk})) f(r_{ij}) f(r_{ik})$$



Angular term:

$$P_m(\cos(\theta)) \quad m = 0, 1, 2, 4, 6$$

(Legendre polynomials)

Radial term:

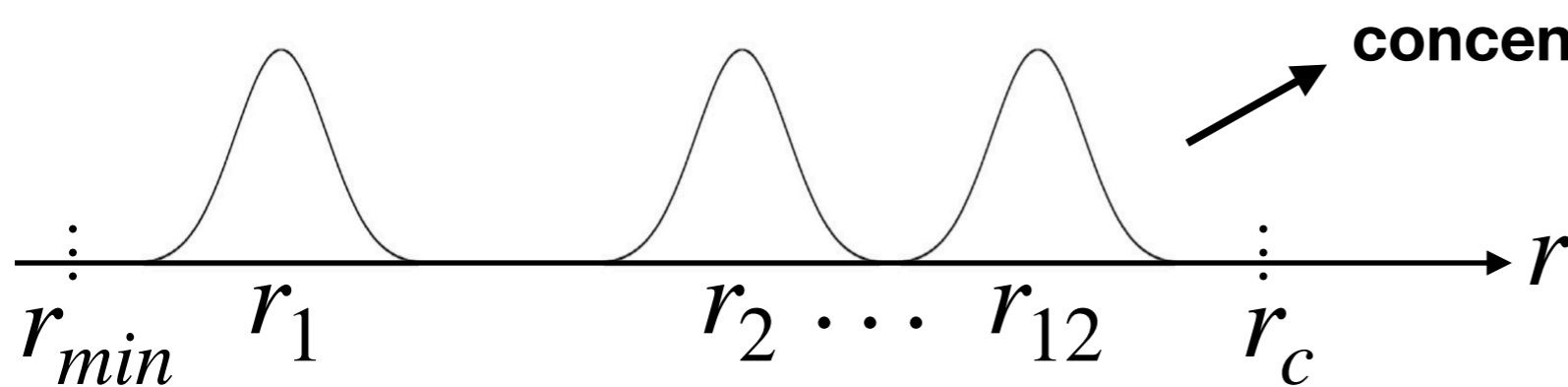
$$f(r) = \frac{1}{r_n} e^{-\frac{(r-r_n)^2}{\sigma^2}} f_c(r)$$

choose 12 values for r_n

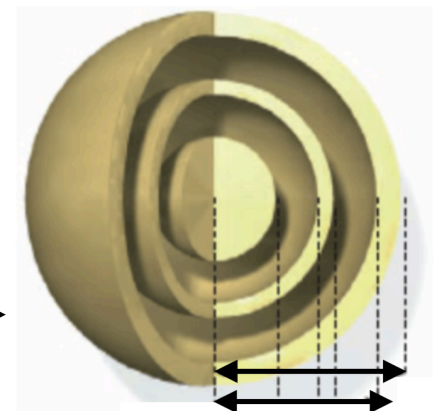
$f_c(r) =$ Cutoff function

G_i 's act as "fingerprints" of the local atomic environment

8 r_n with 5 P_m
 \downarrow
 $G_i^{m,n} = [40 \times 1]$
 for each atom



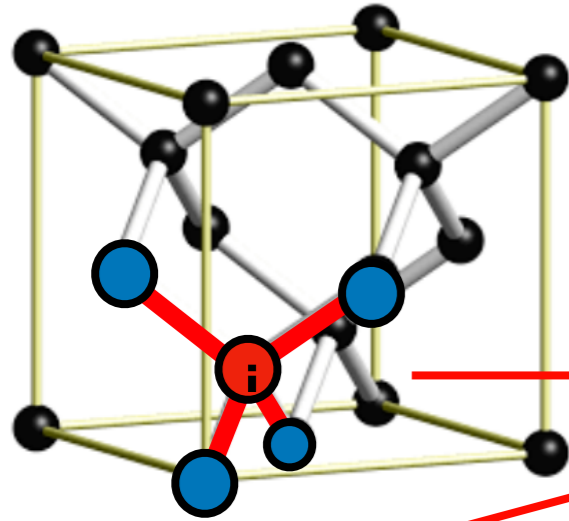
concentric shells



$$f_c(r) = \begin{cases} \frac{(r-r_c)^4}{d^4 + (r-r_c)^4} & r \leq r_c \\ 0, & r \geq r_c \end{cases}$$

PINN potential model

Energy of atom-i



neighboring atom positions
 $(r_1, r_2 \dots r_n)$

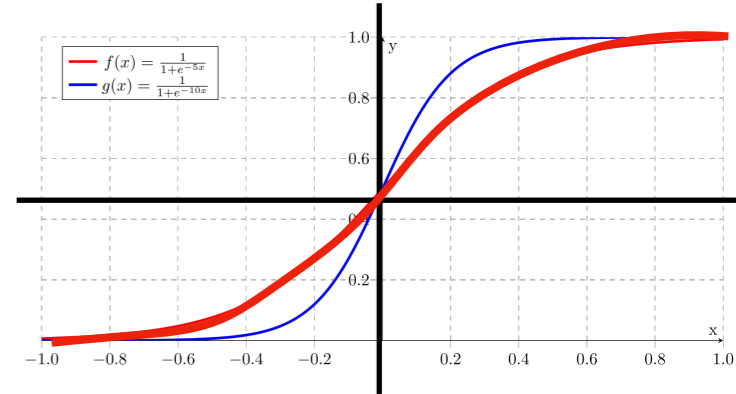
$$G_1 = G_1(r_{ij}, \theta_{ikj})$$

$$G_2 = G_2(r_{ij}, \theta_{ikj})$$

structural "fingerprints"

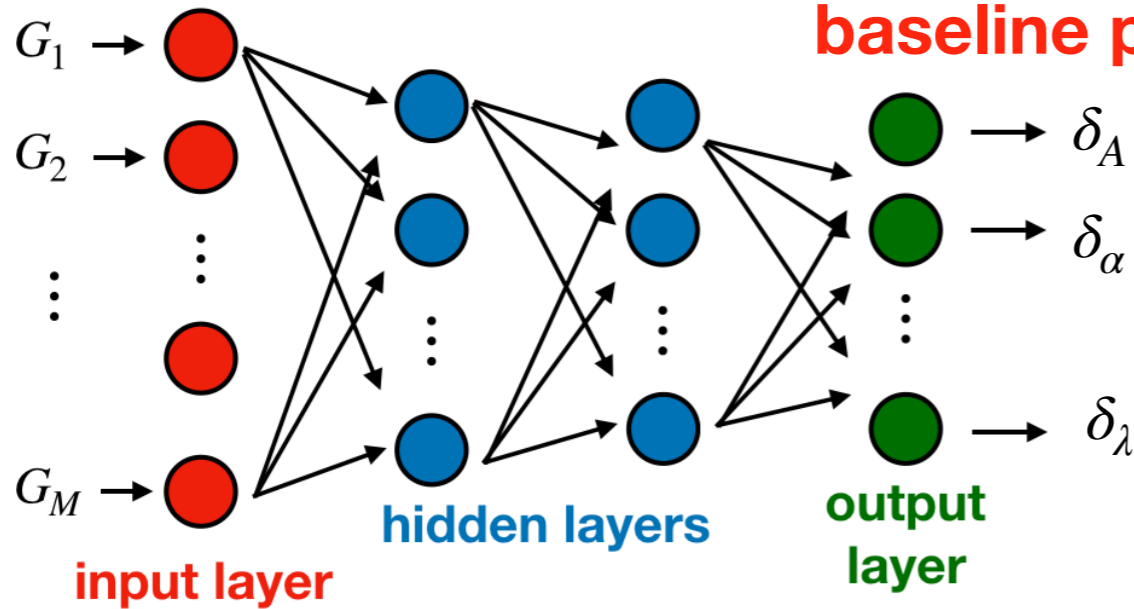
Local structure parameter calculation

$(G_1, G_2 \dots G_M)$



$$A(x) = \frac{1}{e^{-x} + 1} - \frac{1}{2}$$

perturbations to baseline parameters



$(A_o, B_o, \alpha_o, \beta_o, a_o, h_o, \lambda_o, \sigma_o)$ baseline parameters

+
 $(\delta_A, \delta_B, \delta_\alpha, \delta_\beta, \delta_a, \delta_h, \delta_\lambda, \delta_\sigma)$ NN perturbation

=
 $(A_i, B_i, \alpha_i, \beta_i, a_i, h_i, \lambda_i, \sigma_i)$ atom-i parameterization

Training:

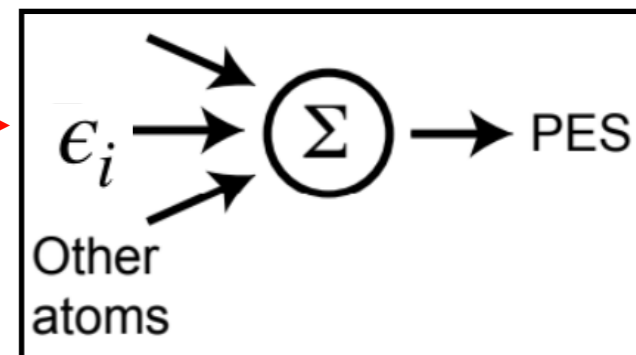
$$RMSE = \left(\frac{\sum_s (E_s - \tilde{E}_s)^2}{N} \right)^{\frac{1}{2}} = NN(w, b)$$

Find the NN weights and bias's which minimize the RMSE

BOP Model

atomic energy $\rightarrow \epsilon_i$

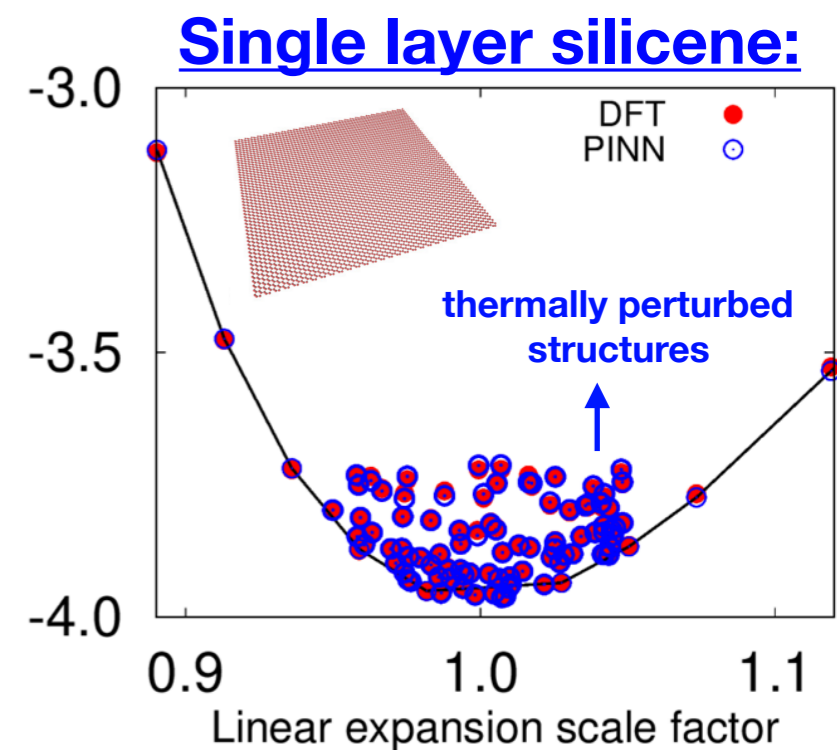
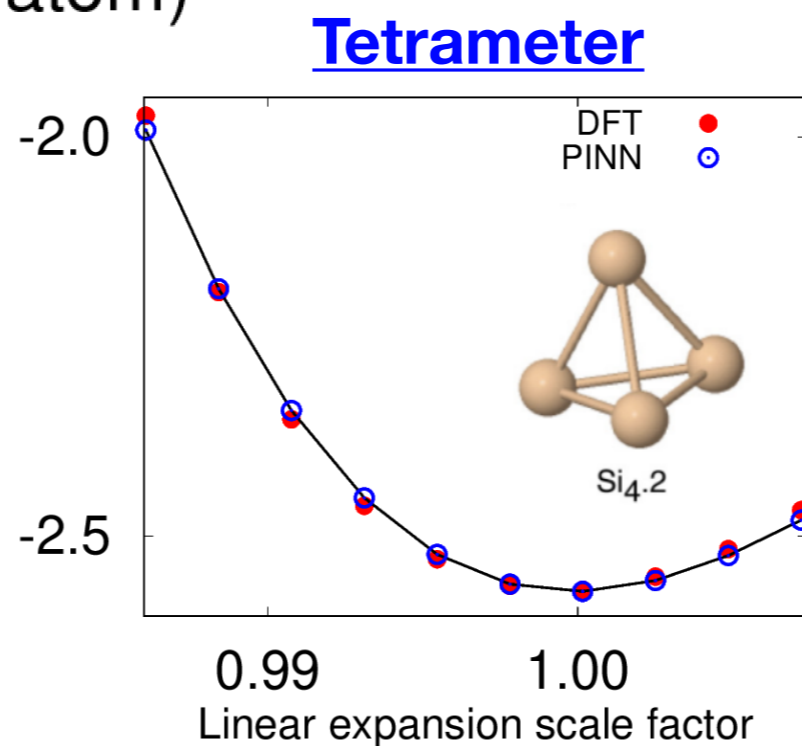
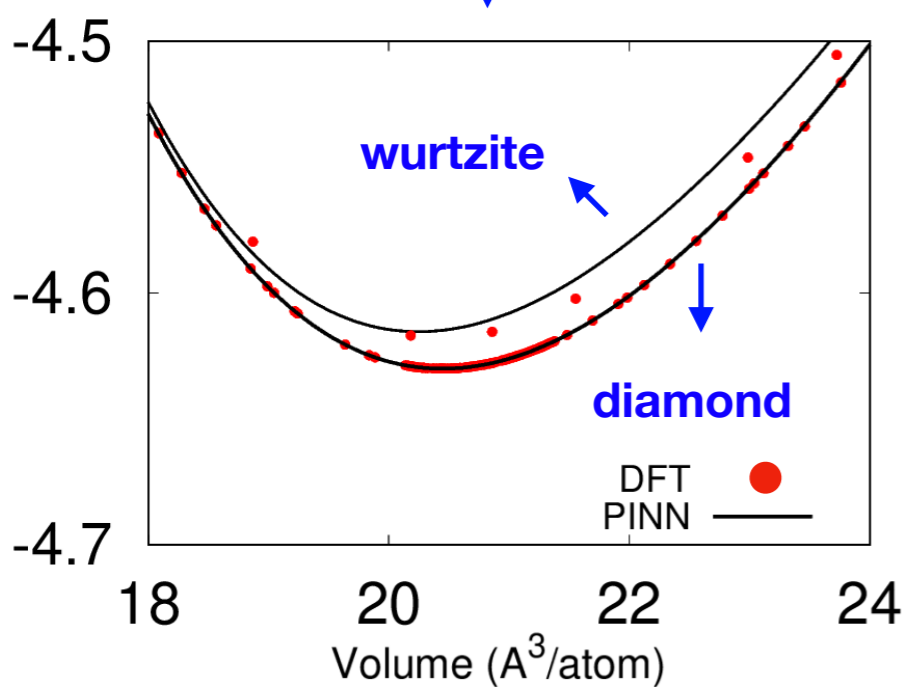
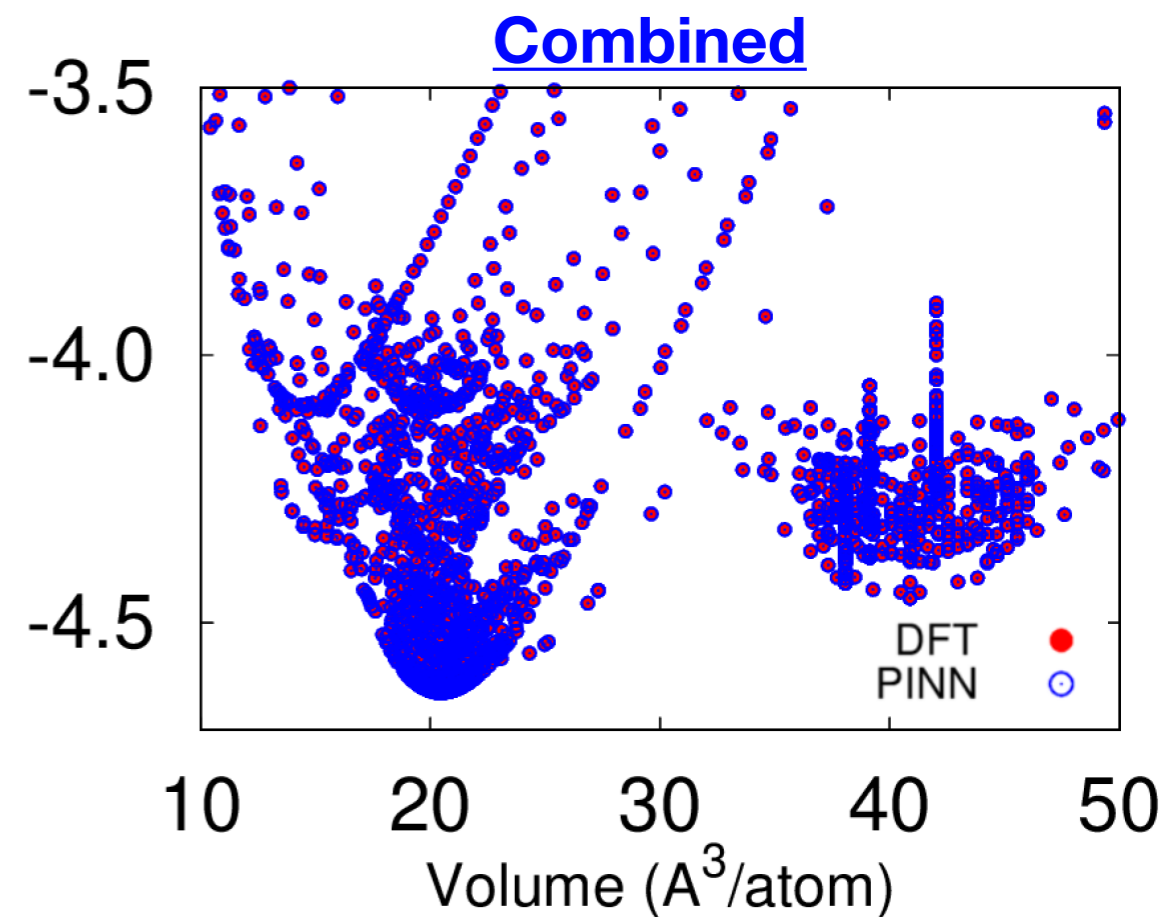
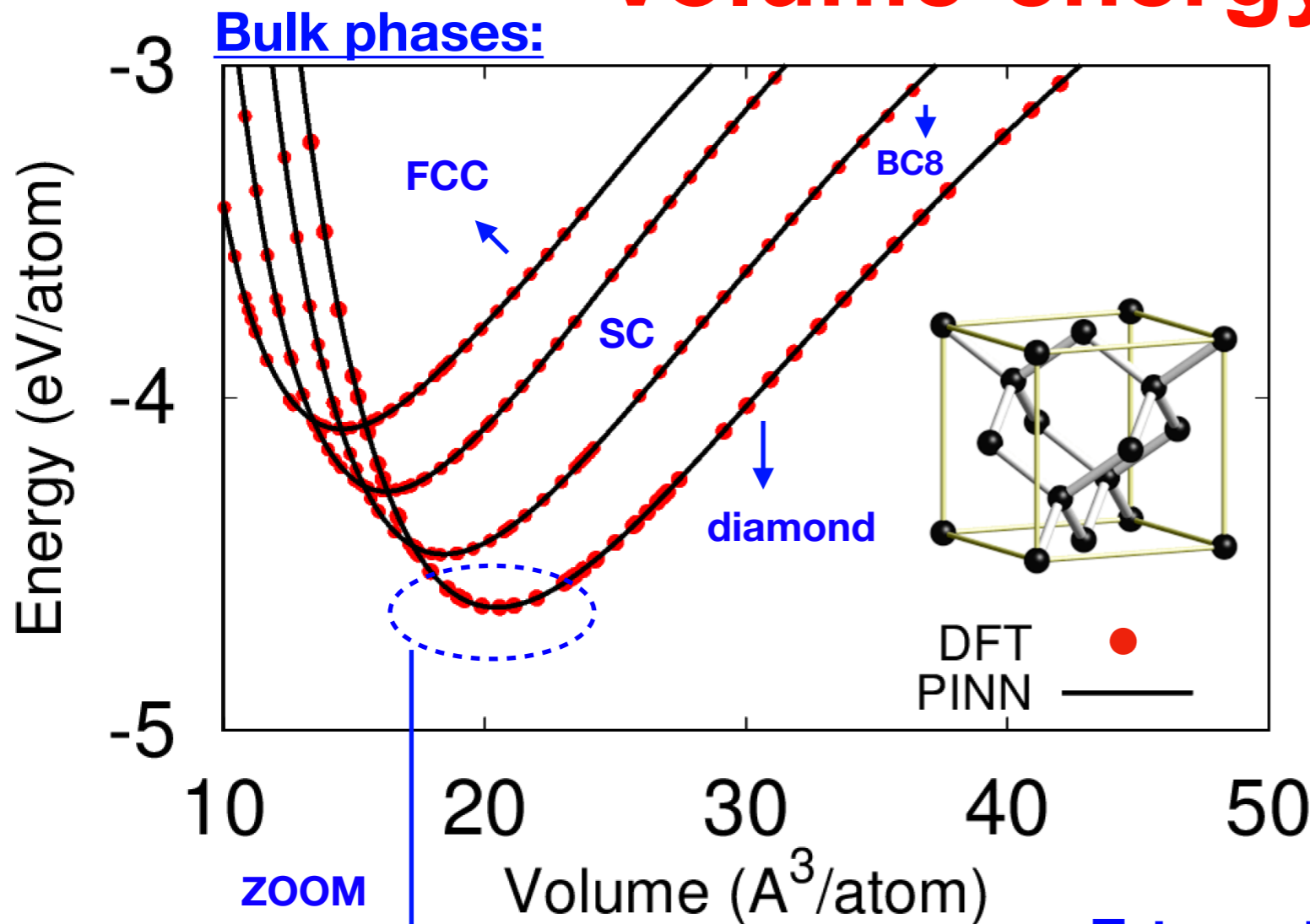
Structure's energy



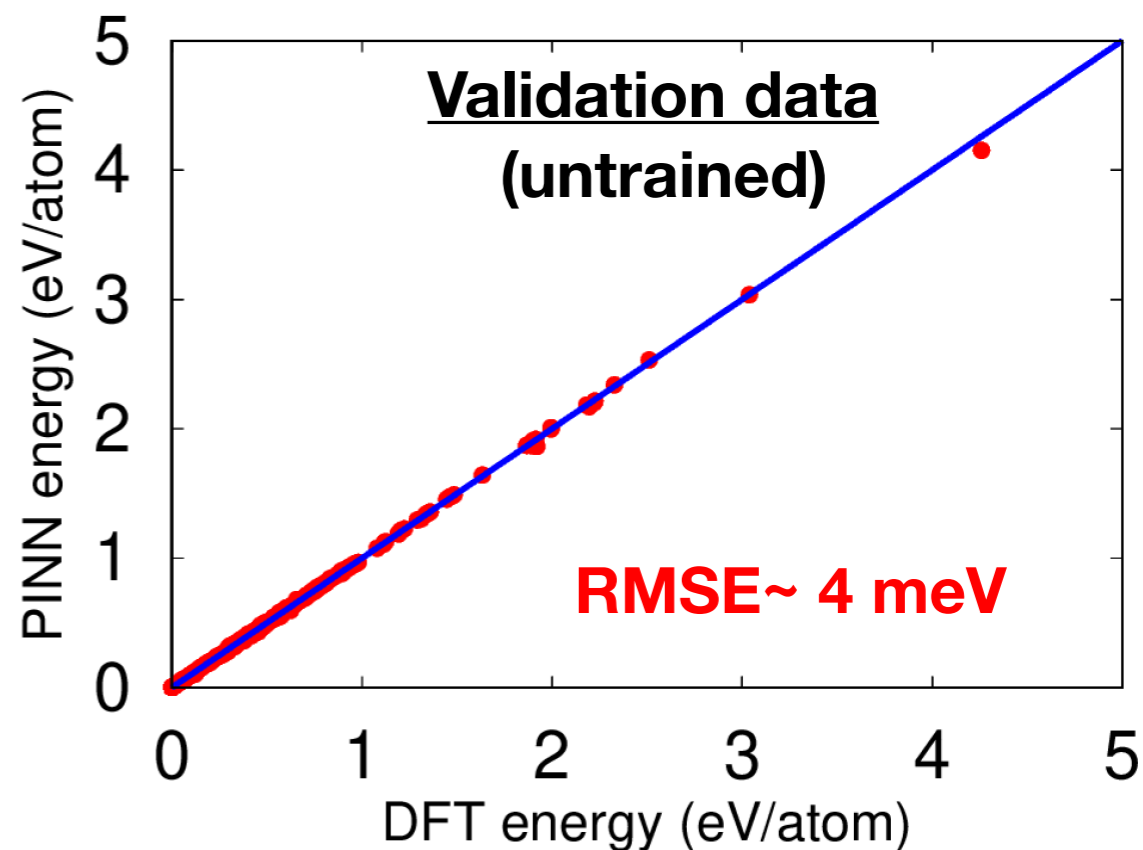
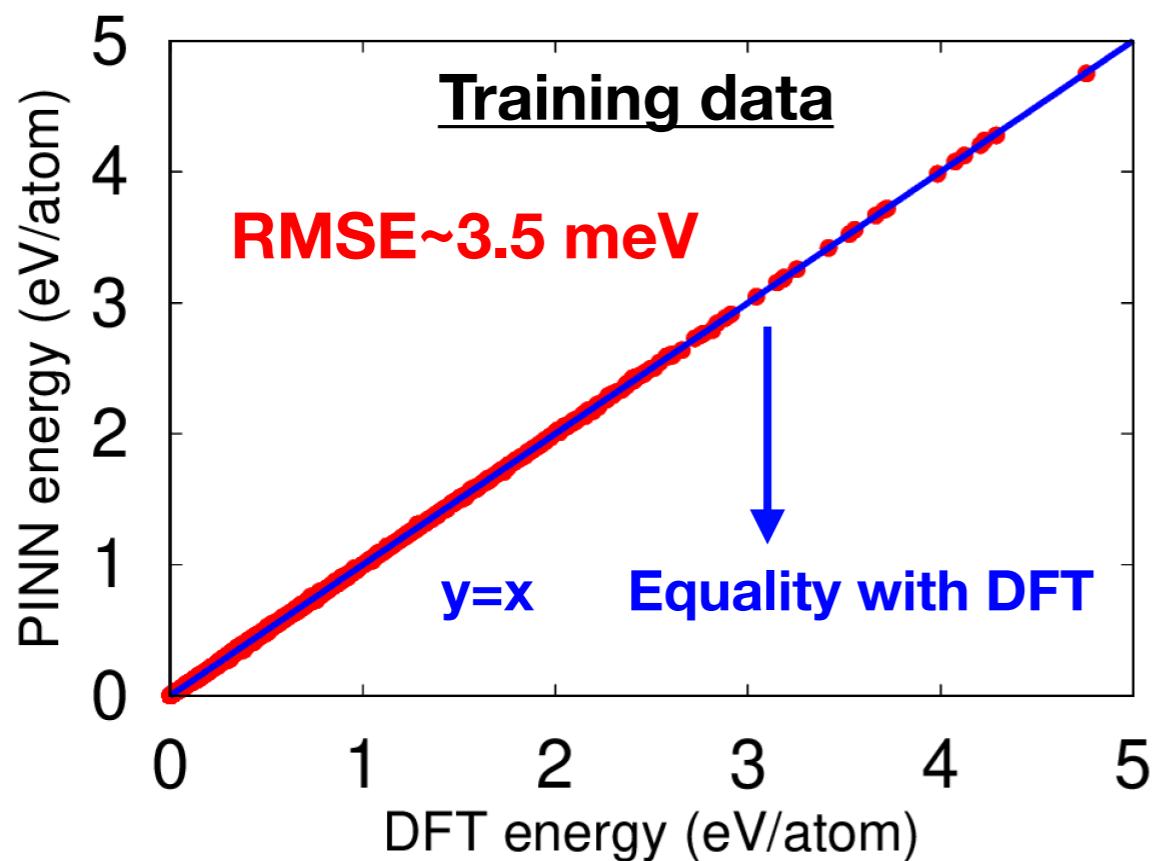
Results

Volume-energy curves

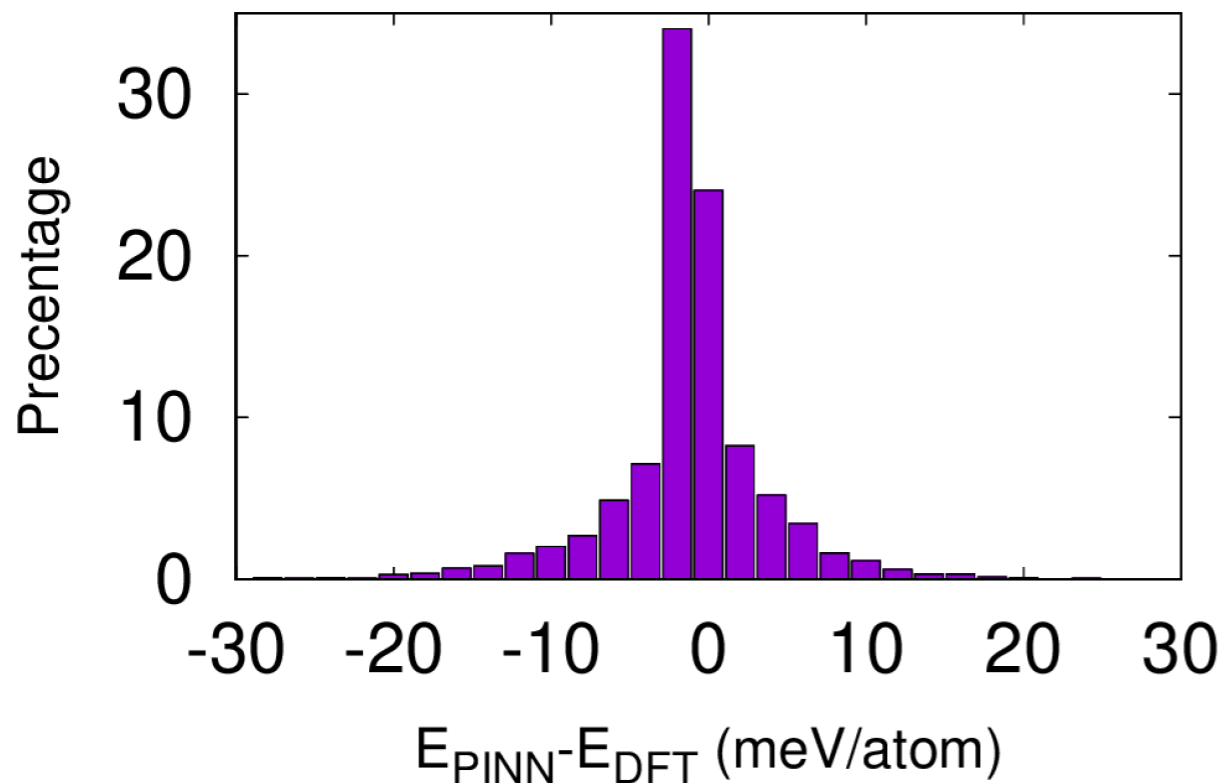
NN: 60x32x8



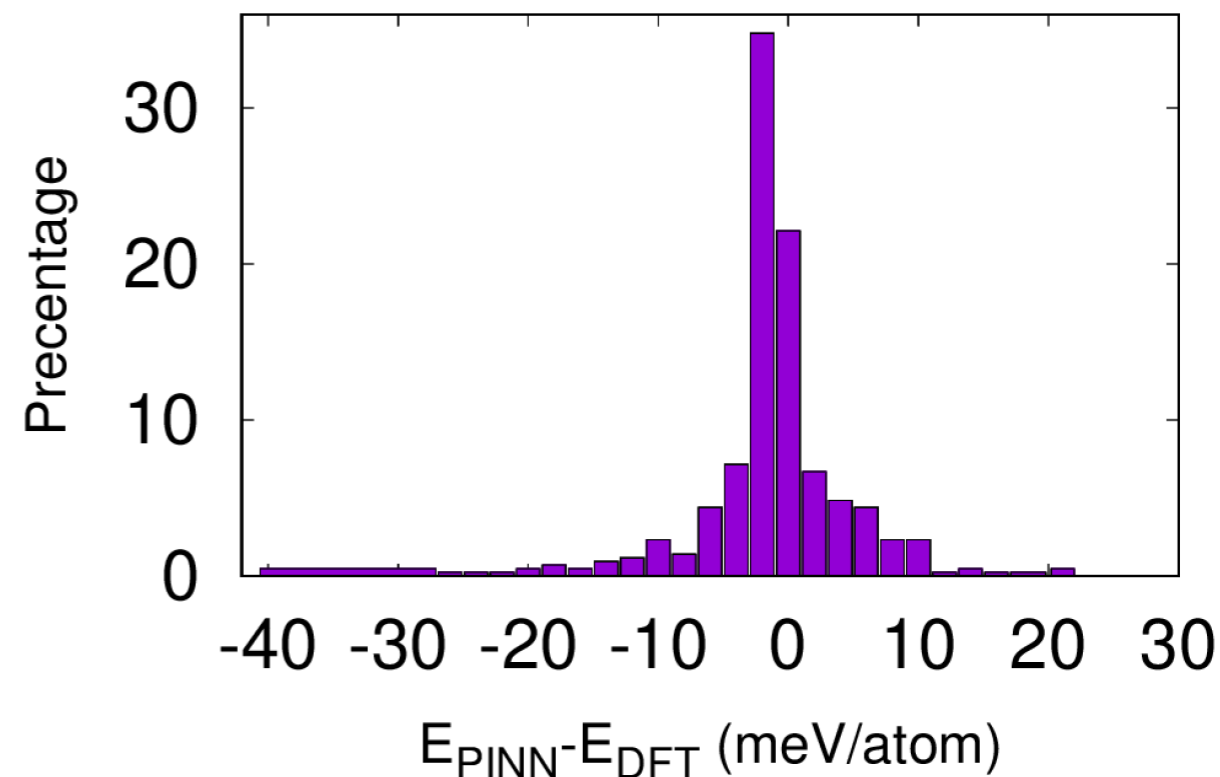
Reproduction of DFT energy landscape



Distribution of training error

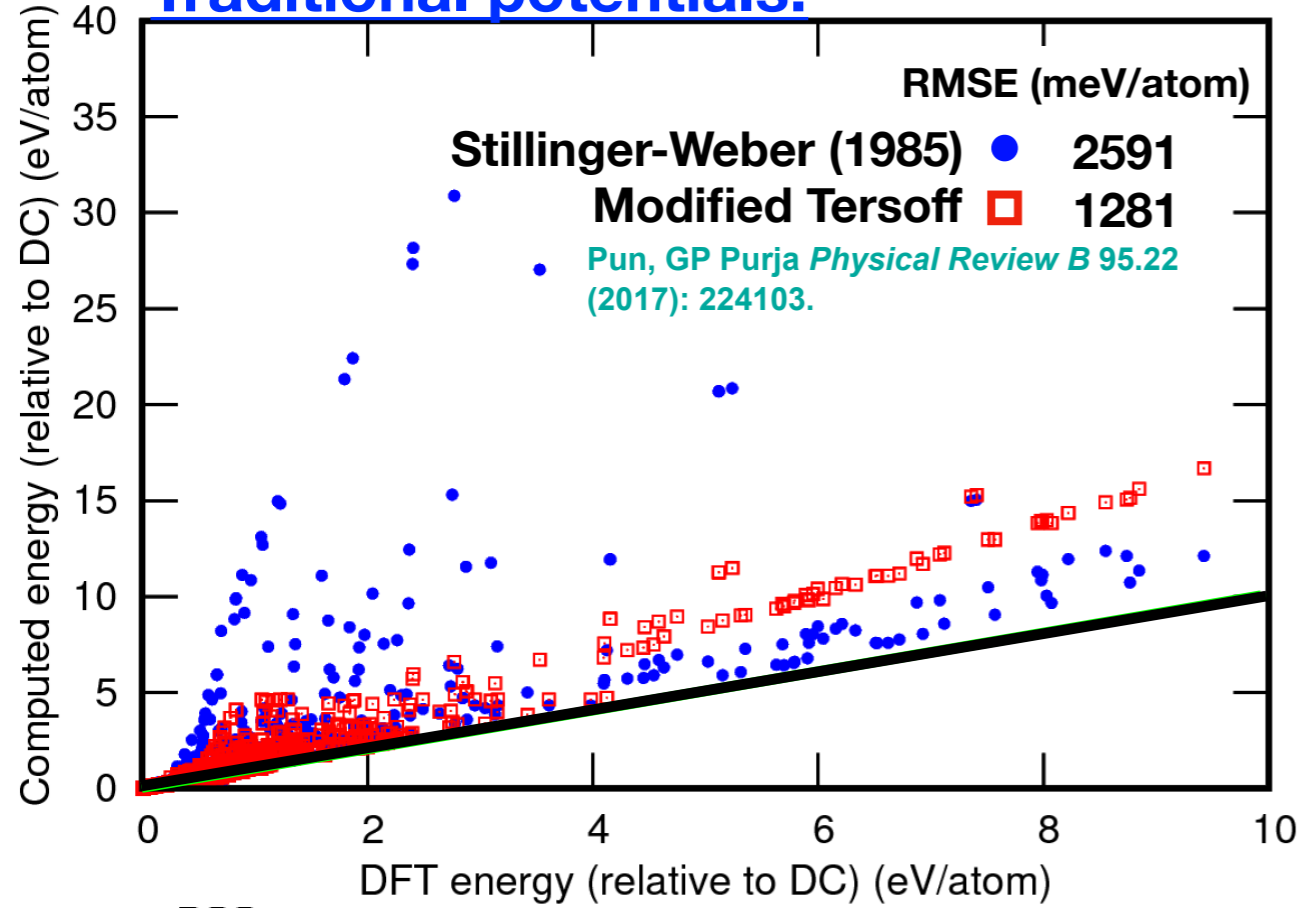


Distribution of validation error

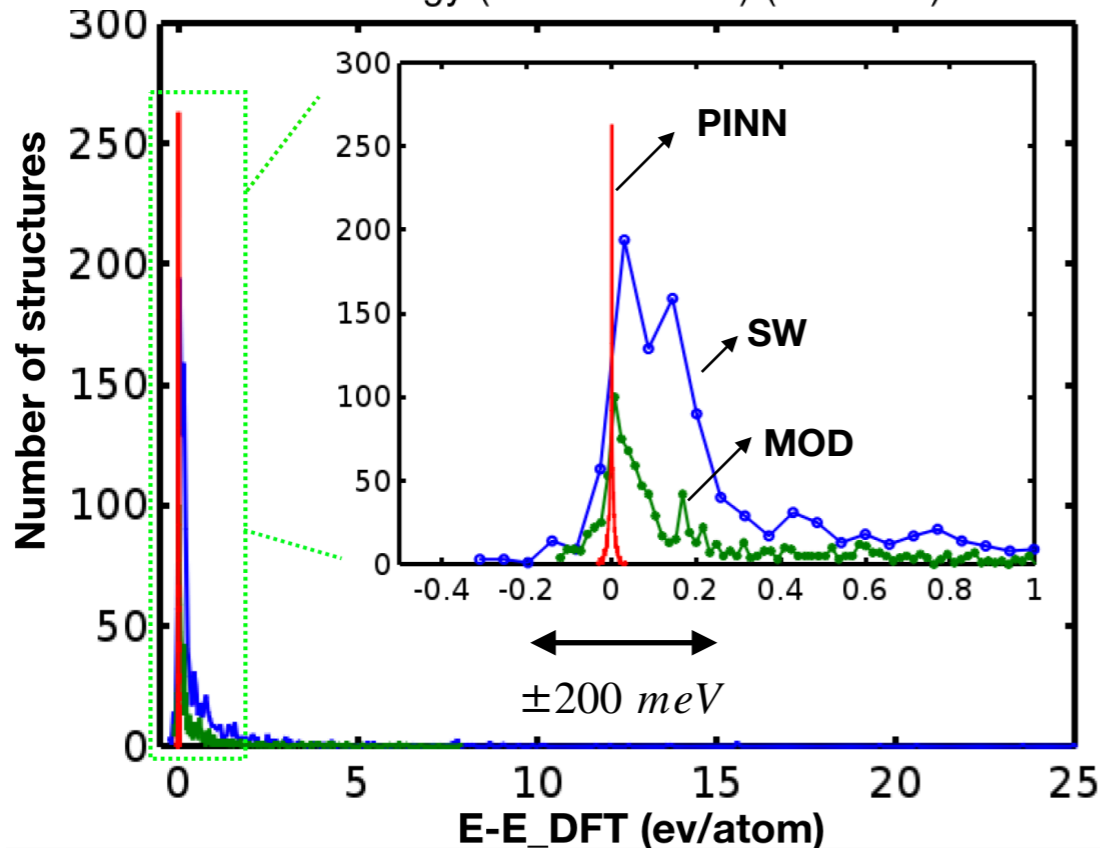
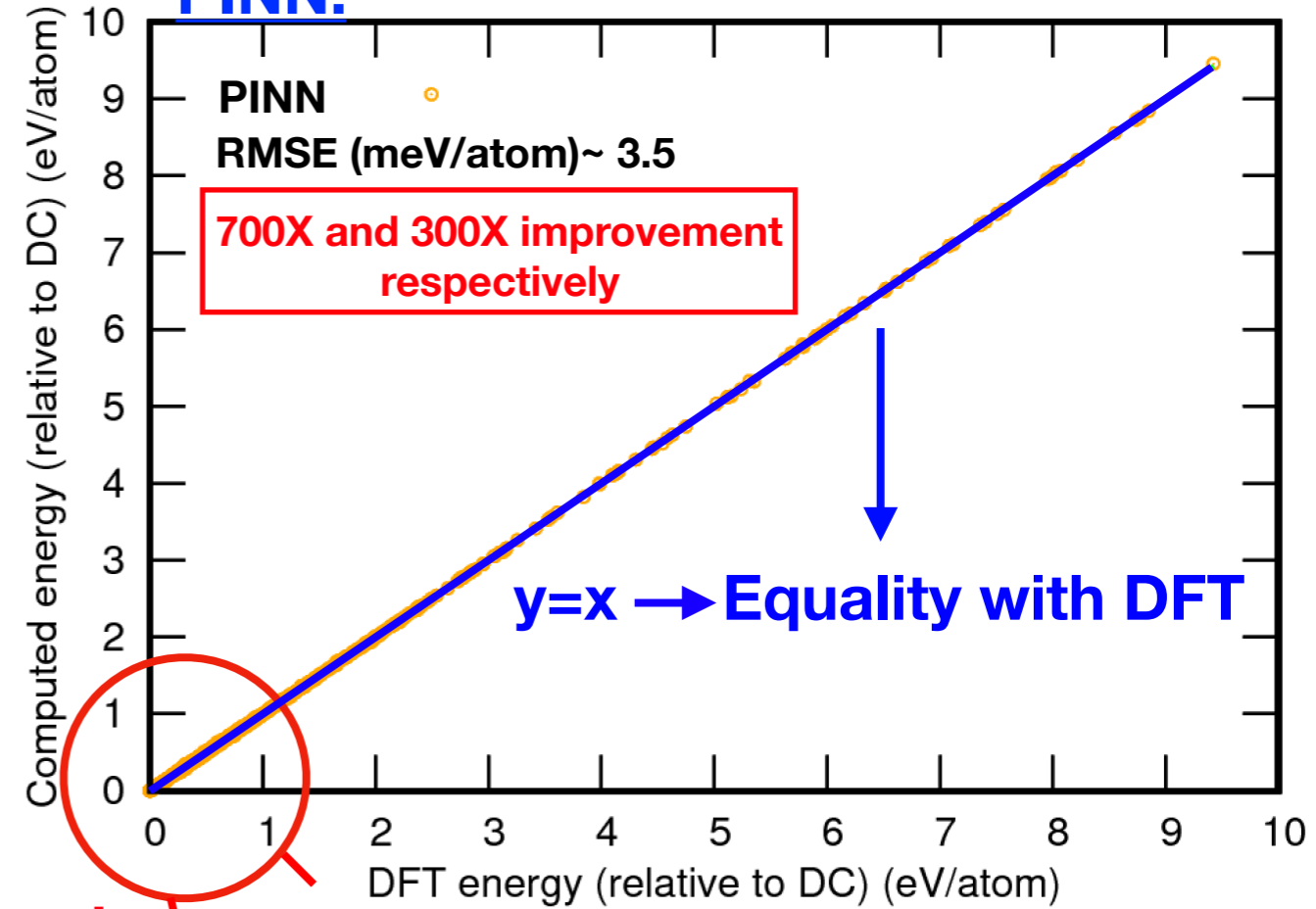


Si PINN comparison with traditional potential

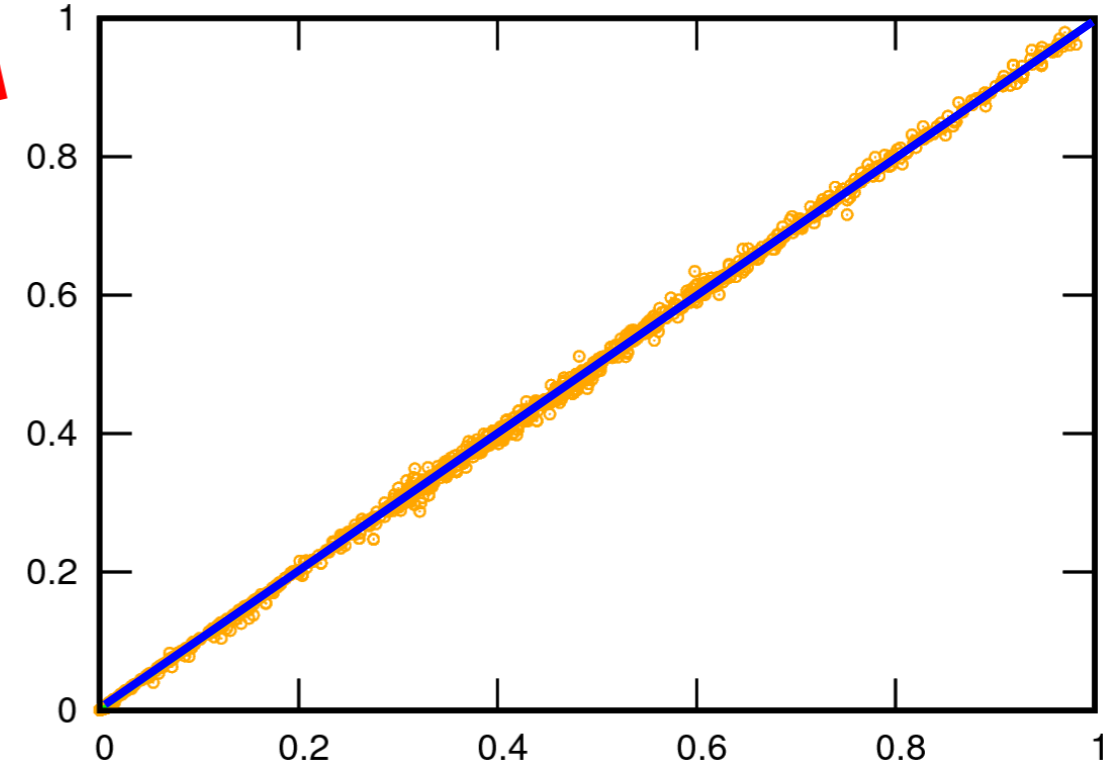
Traditional potentials:



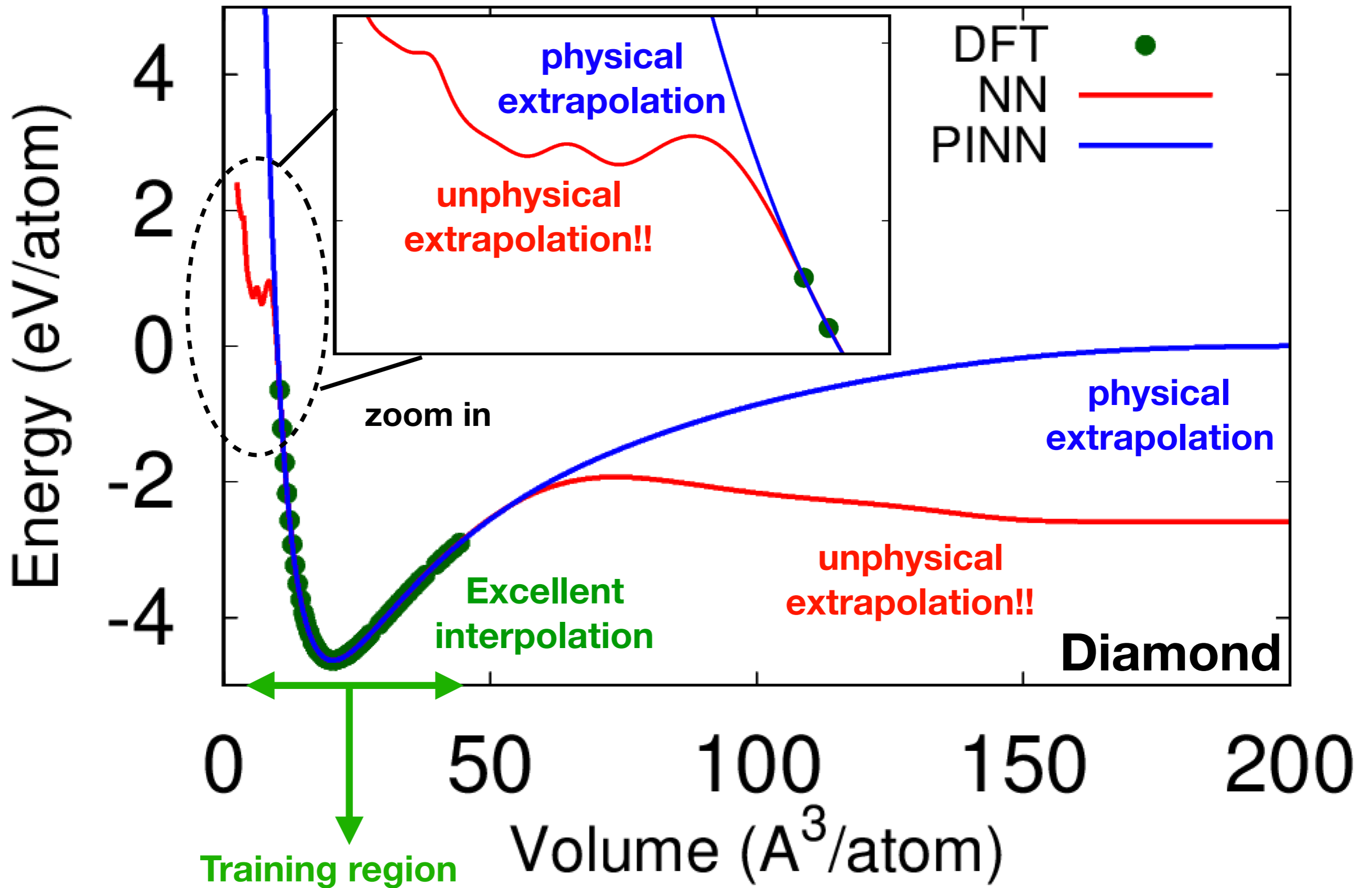
PINN:



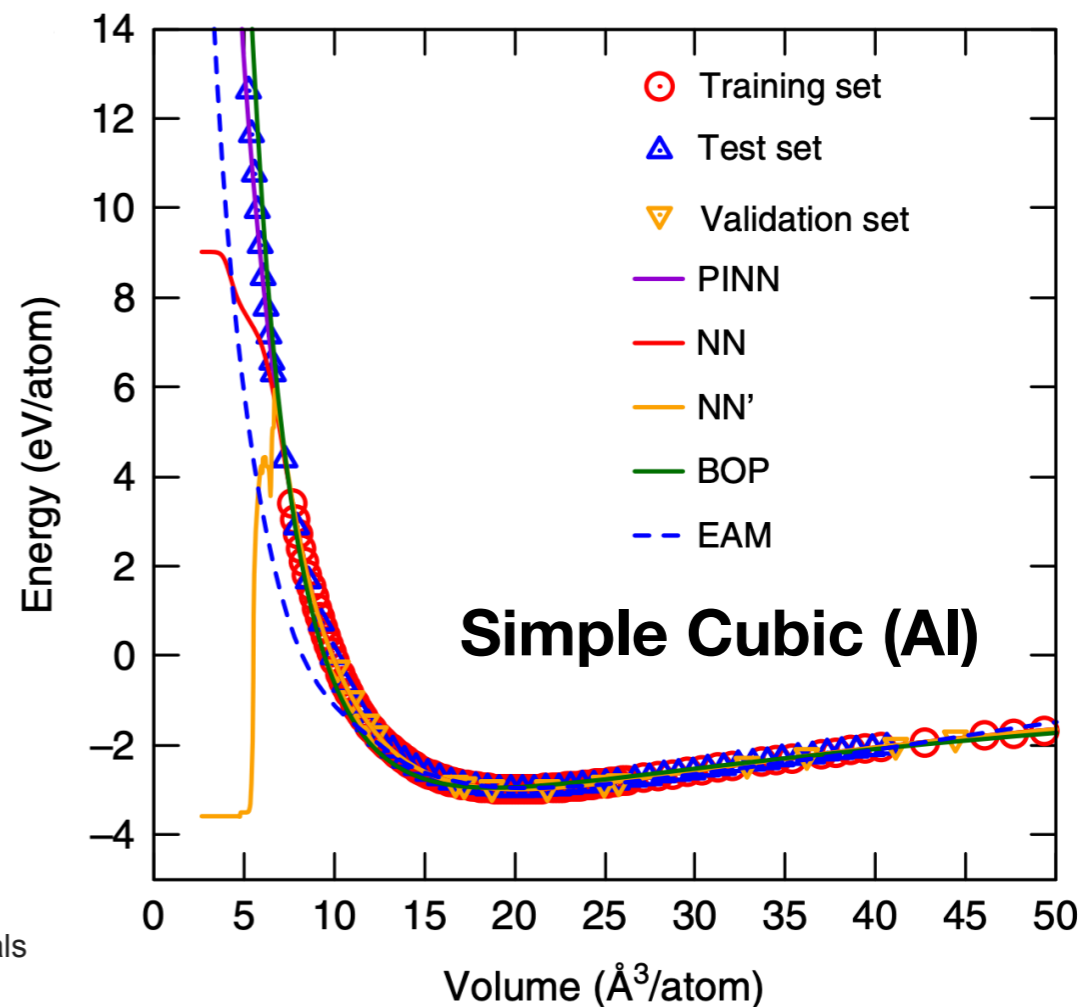
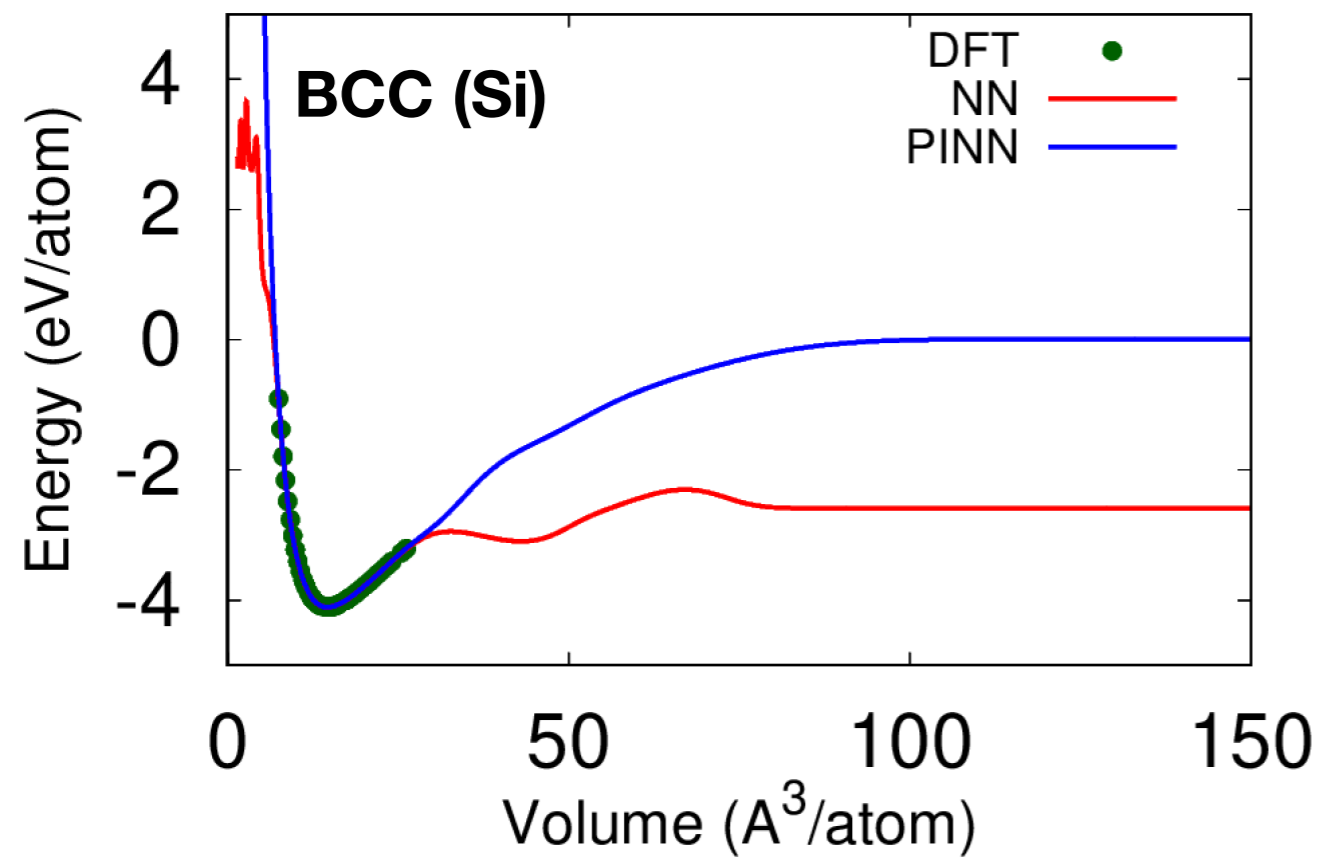
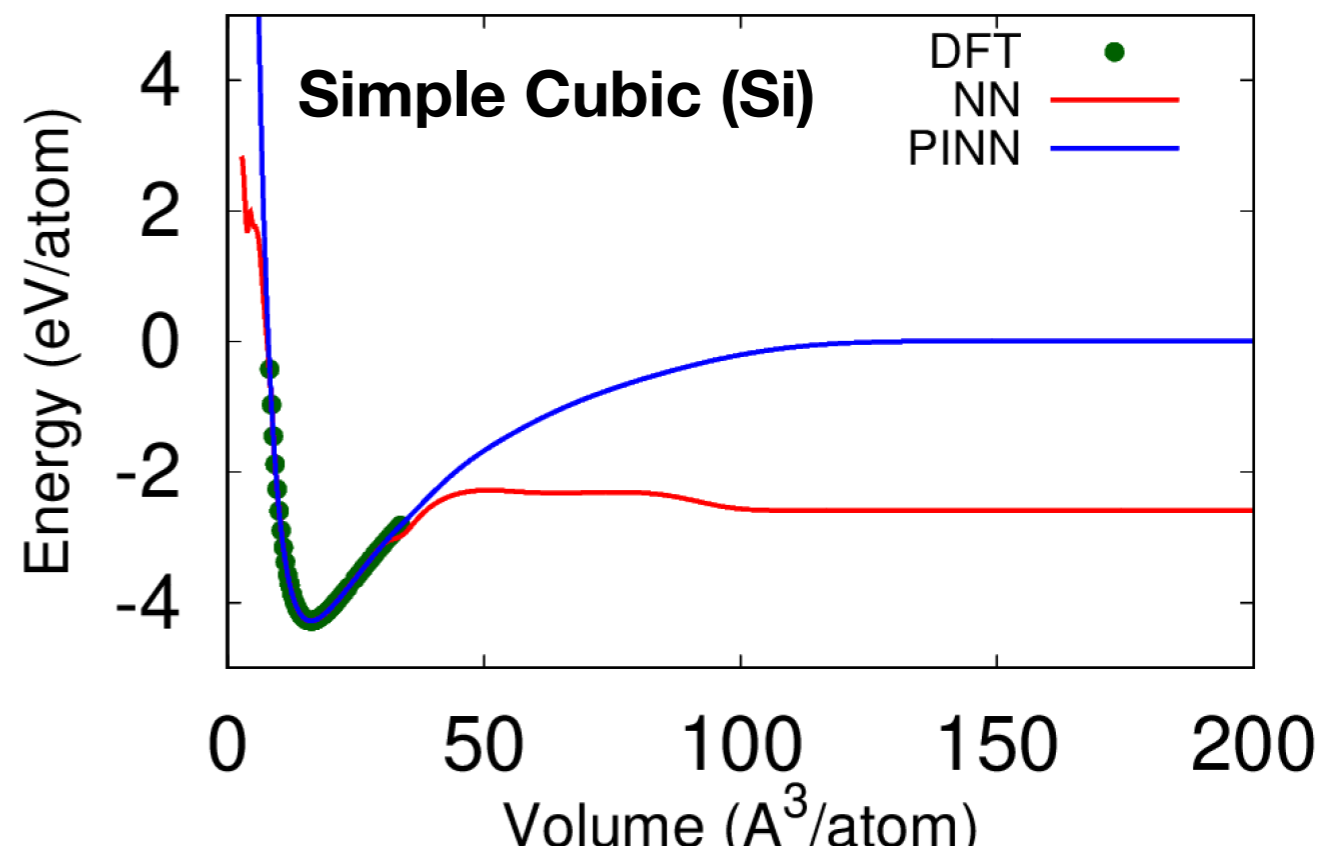
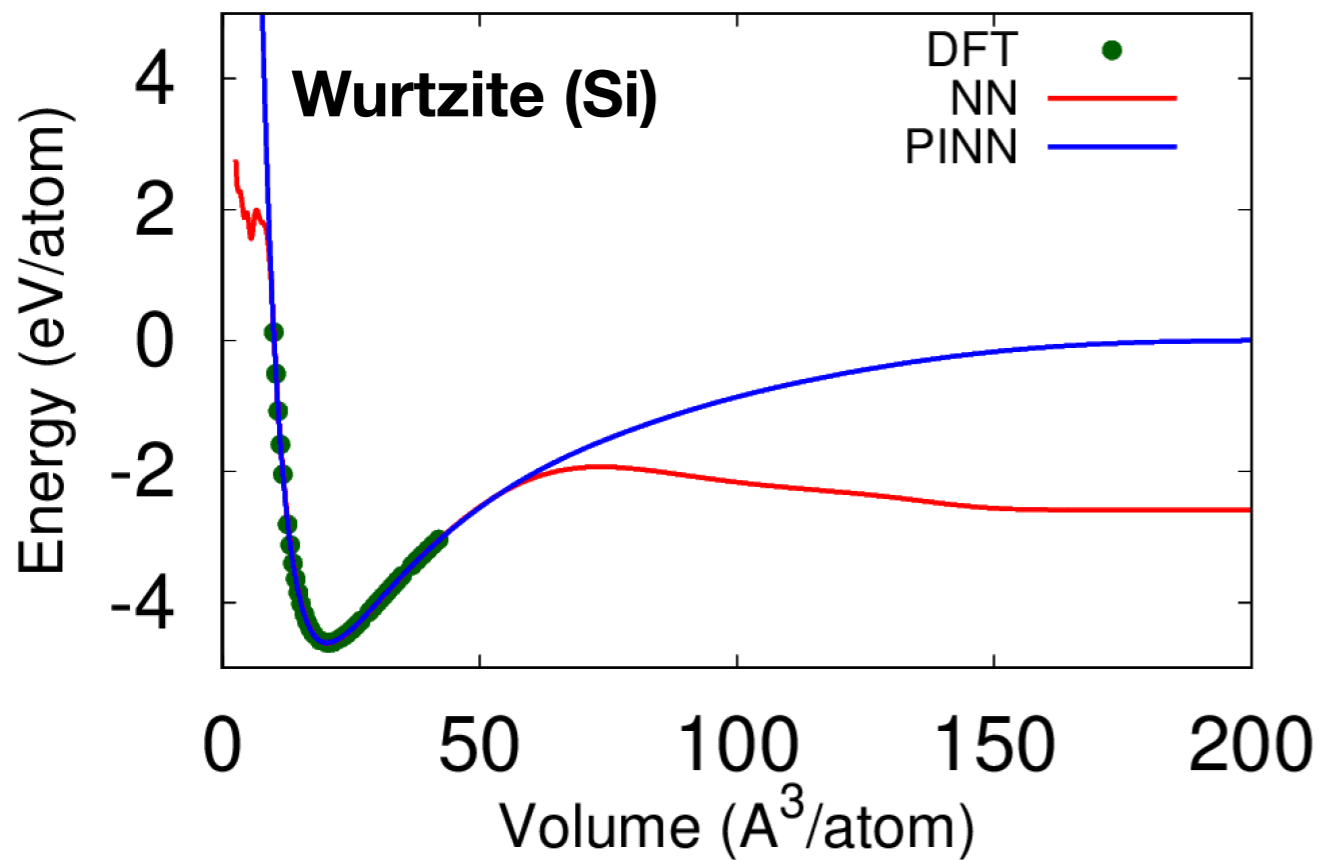
zoom in



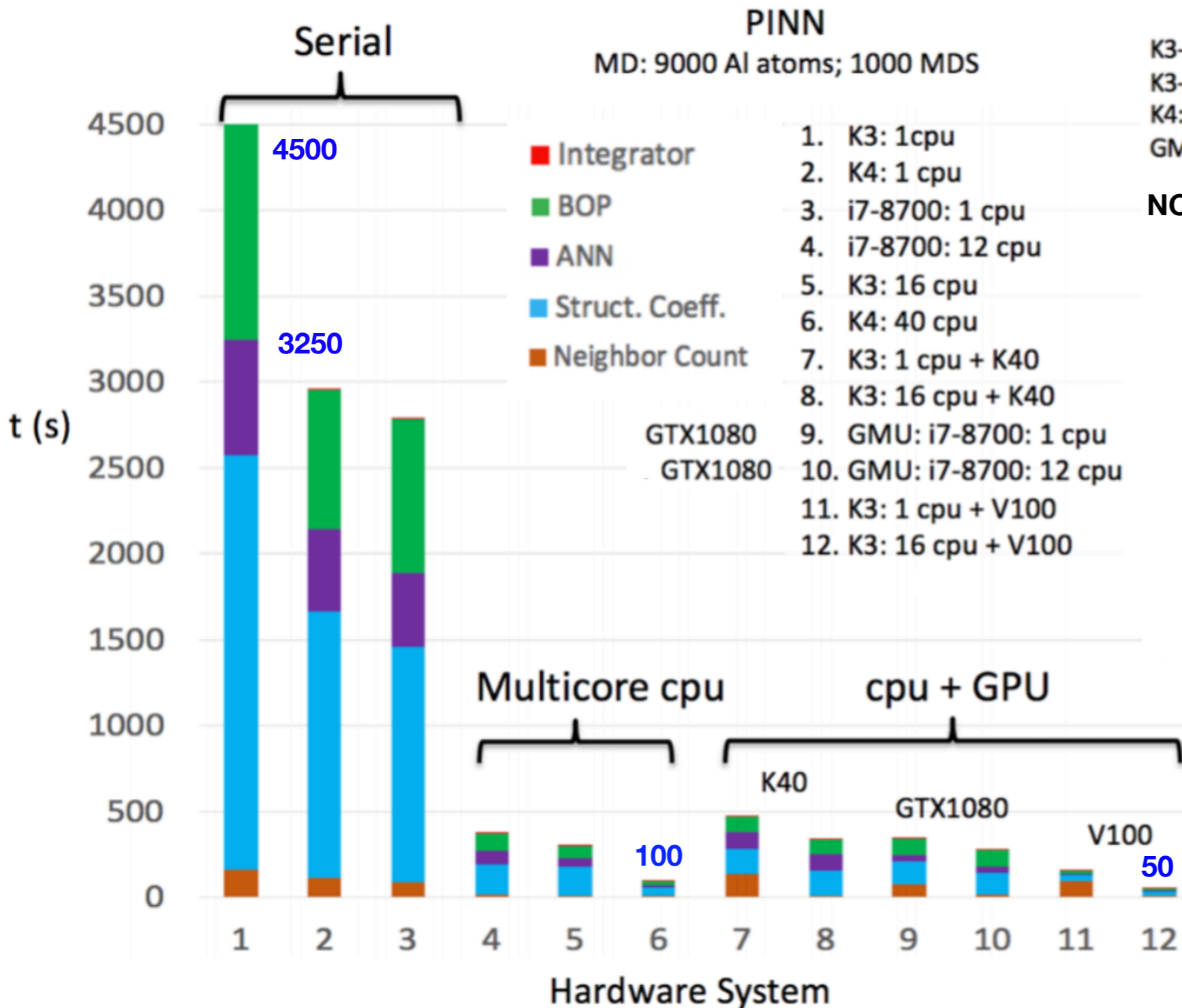
ANN vs PINN Extrapolation



ANN vs PINN Extrapolation

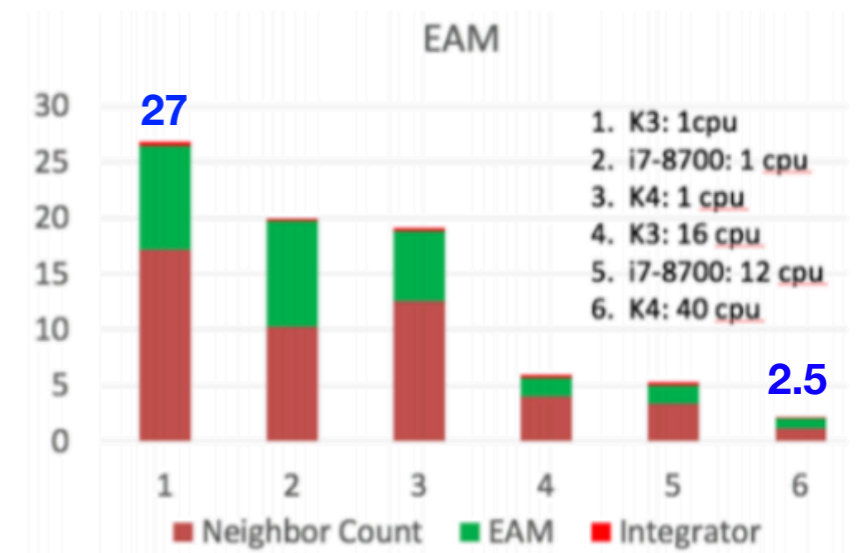


Computational efficient



K3-K40: SGI ICE Altix X 16 core cpu + NVidia Kepler K40 GPU
 K3-V100: SGI ICE Altix X 16 core cpu + NVidia Volta V100 GPU
 K4: Intel 6148 Skylake 40 core cpu
 GMU: Intel i7-8700 CPU 12 core cpu + NVidia GTX1080 GPU

NOTE: results sensitive to hyper parameter choice



PINN vs EAM

serial: 166X slower
 40 CPU: ~40X slower
 GPU: ~20X slower

NN vs EAM

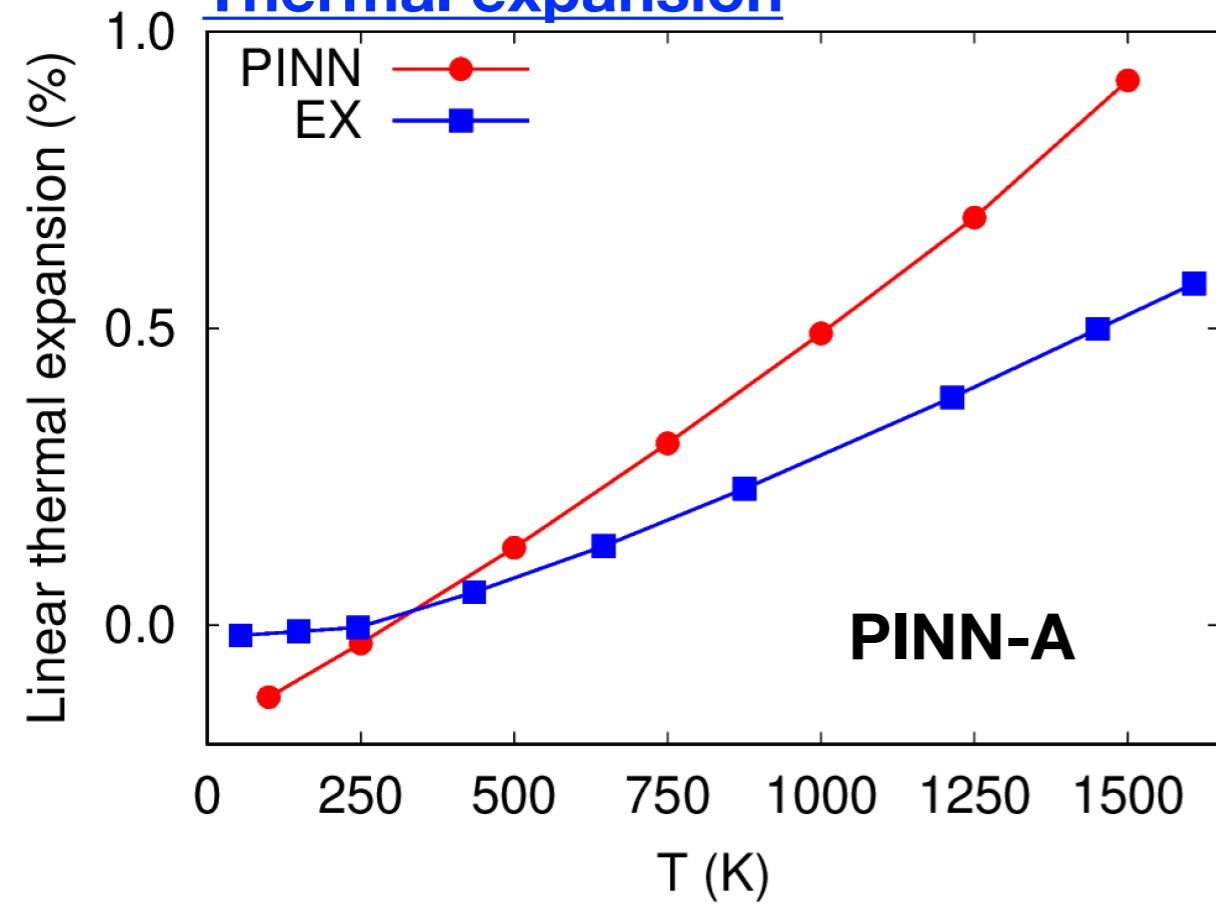
serial: 120X slower
 40 CPU: ~30X slower
 GPU: ~15X slower

Source: Vesselin Yamakov (NASA)

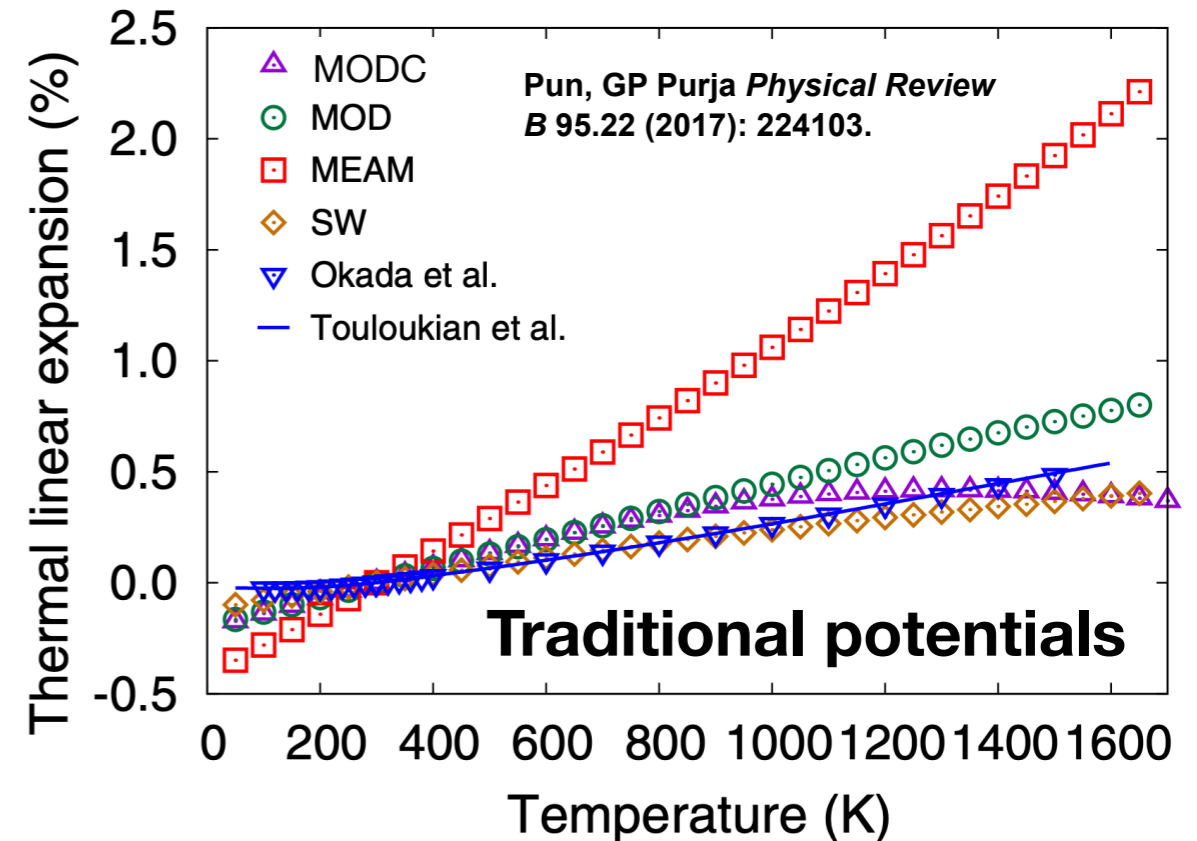
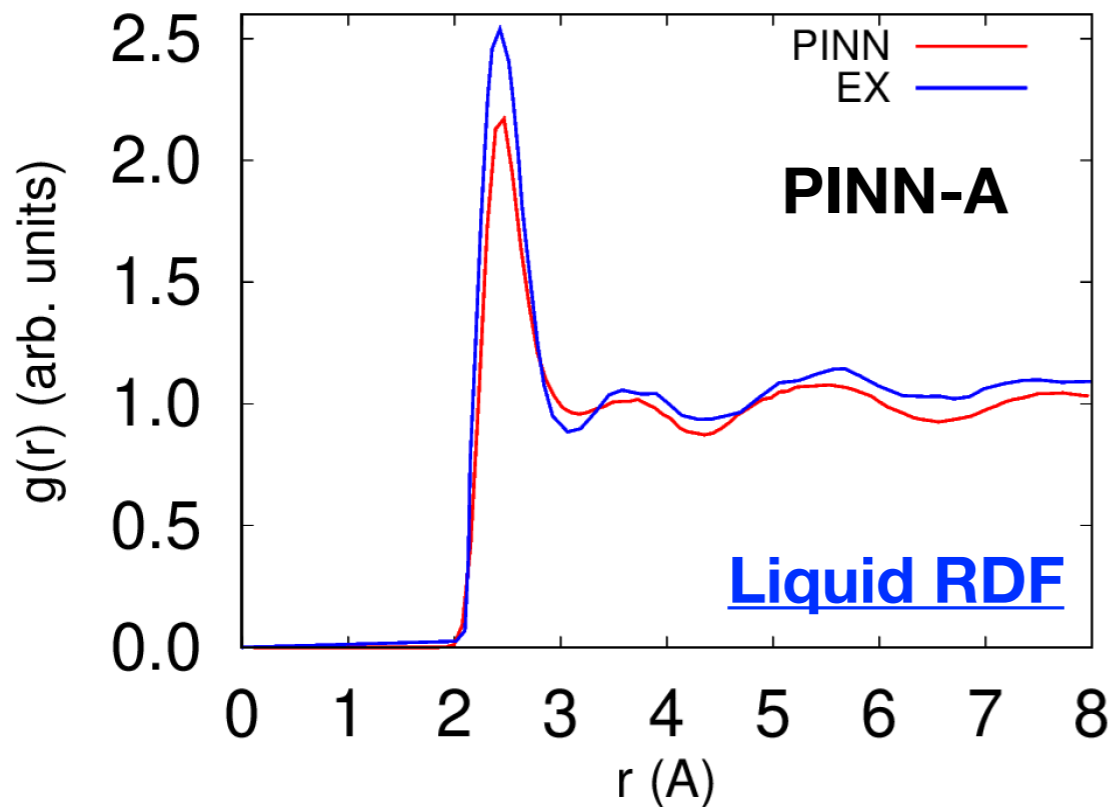
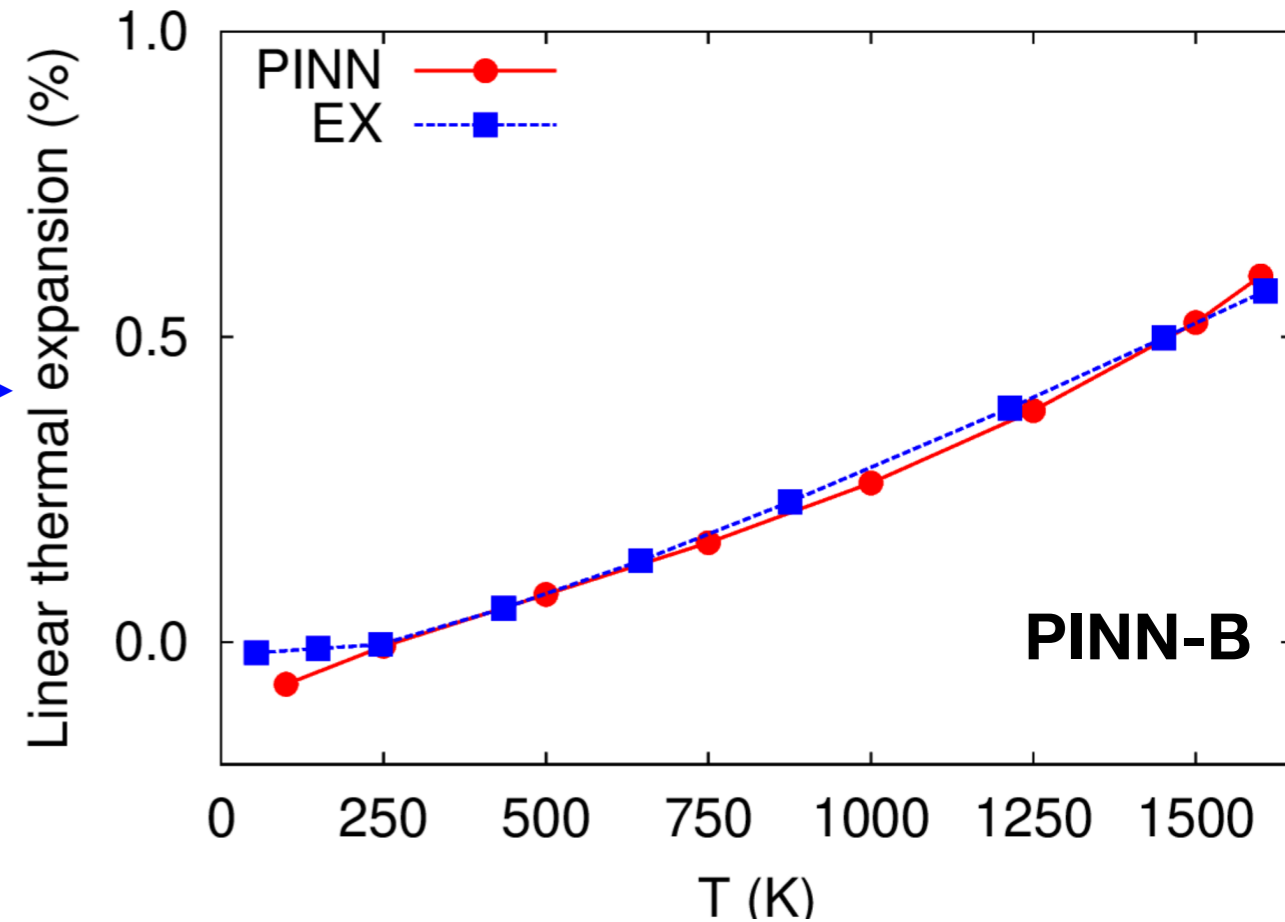
Paragrand MC: <https://software.nasa.gov/software/LAR-18773-1>

Thermal properties (preliminary)

Thermal expansion



systematic improvement



Future work

- **Get PINN_BOP working in LAMMPS**
- **Hyper-parameter tune (speed/accuracy)**
- **Explore other Chemical systems:**
 - **Ge, Pt, Cu**
- **Binary: SiGe, SiAl, etc**
- **Possibly explore other traditional potentials formats**
 - **(PINN_EAM, PINN_ADP, etc)**
- **Applications**
 - **study thermal properties of 2D structures**
 - **Si ,Ge, SiGe**

Conclusions

- **Developed a new silicon interatomic potential using the new PINN potential format**
- **Even in preliminary stage we are obtaining excellent agreement with the DFT energies**
- **Current potential reproduces DFT data around ~500x better than current traditional potentials**
- **Better transferability than ANN potentials**
- **Investigating methodological considerations to streamlining the fitting procedure for faster future development**

Acknowledgements

- Ganga Purja Pun
- Vesselin Yamakov
- Francesca Tavazza
- Yuri Mishin
- Adam Robinson
- GMU, NRC, NIST



References:

- Stillinger, Frank H., and Thomas A. Weber. "Computer simulation of local order in condensed phases of silicon." *Physical review B* 31.8 (1985): 5262.
- ("In review") G. P. Purja Pun(1), R. Batra(2), R. Ramprasad (3) and Y. Mishin(1): (1) George Mason Univ., (2) Univ. Connecticut, (3) Georgia Tech
- Pun, GP Purja, and Y. Mishin. "Optimized interatomic potential for silicon and its application to thermal stability of silicene." *Physical Review B* 95.22 (2017): 224103.

Image references:

- https://www.tf.uni-kiel.de/matwis/amat/iss/kap_5/backbone/r5_3_3.html
- <https://www.wordstream.com/blog/ws/2017/07/28/machine-learning-applications>
- https://www.researchgate.net/figure/268158499_fig1_Figure-1-Phase-diagram-of-SiGe-alloys-showing-separation-of-the-solidus-and-liquidus
- <http://evolution.skf.com/us/bearing-research-going-to-the-atomic-scale>
- <https://www.chegg.com/homework-help/questions-and-answers/consider-concentric-metal-sphere-spherical-shells-shown-figure--innermost-solid-sphere-rad-q4808250>