TURBULENCE

Mathematics or Physics Problem?

LAMMPS Workshop and Symposium

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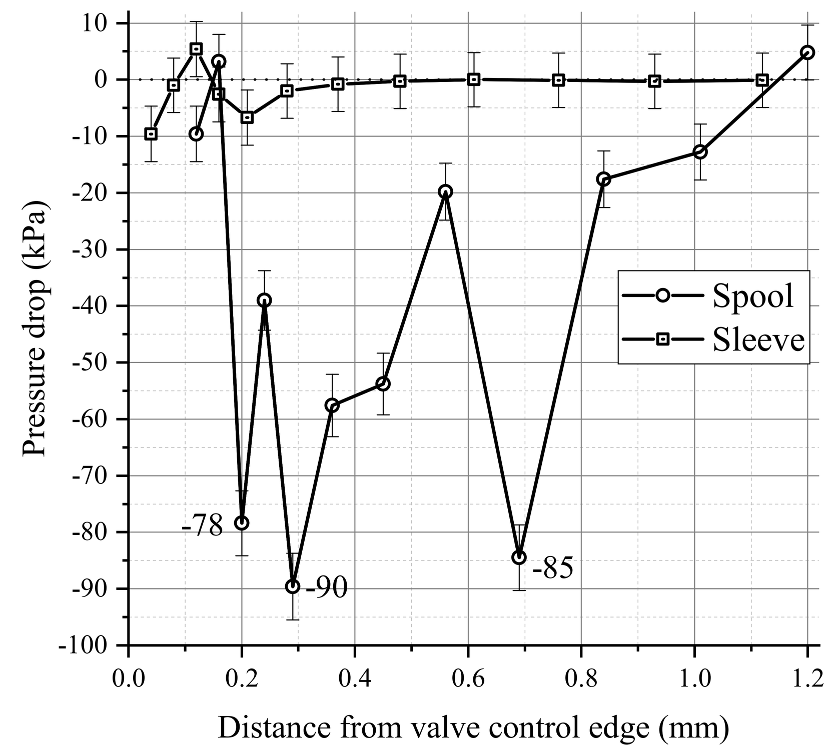
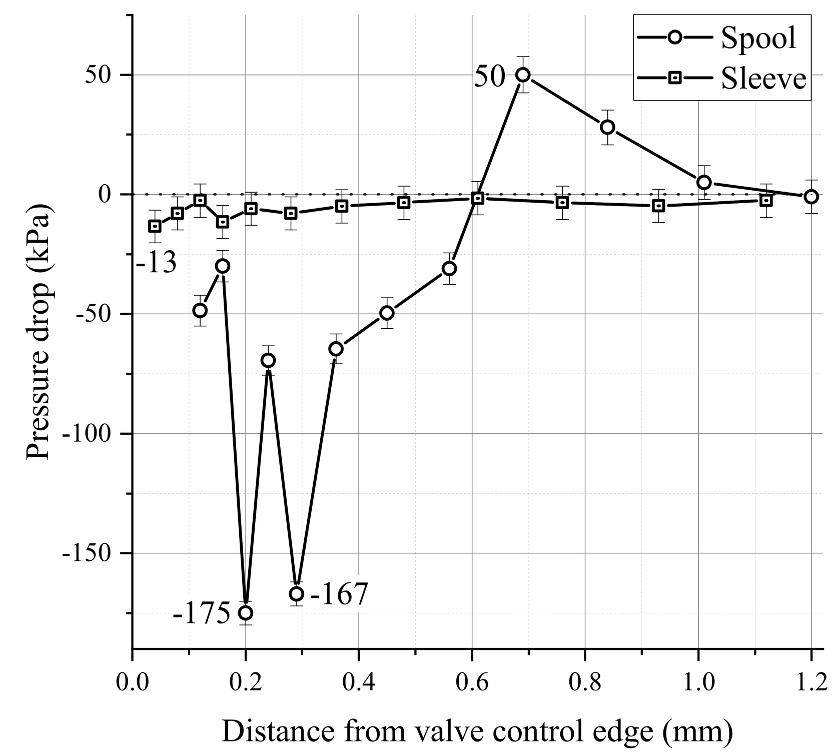
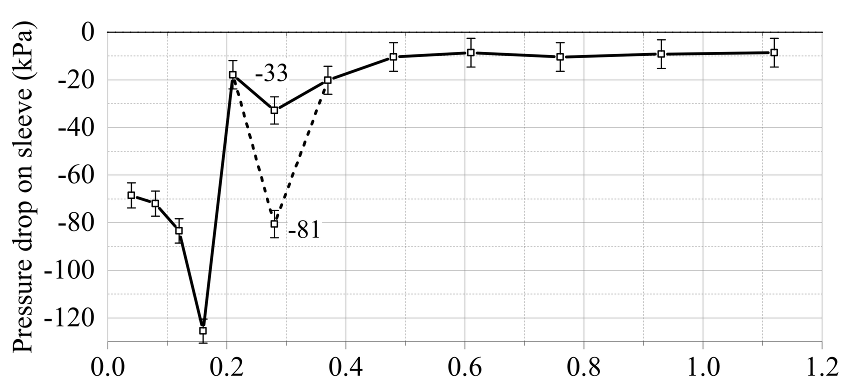
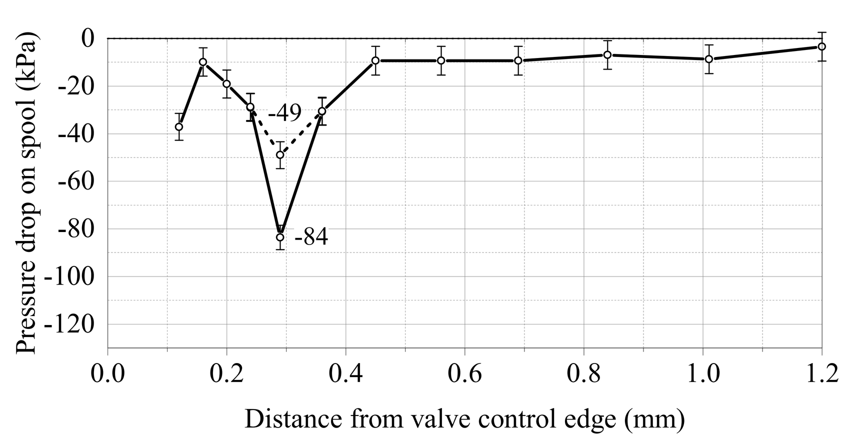
Jan Lugowski

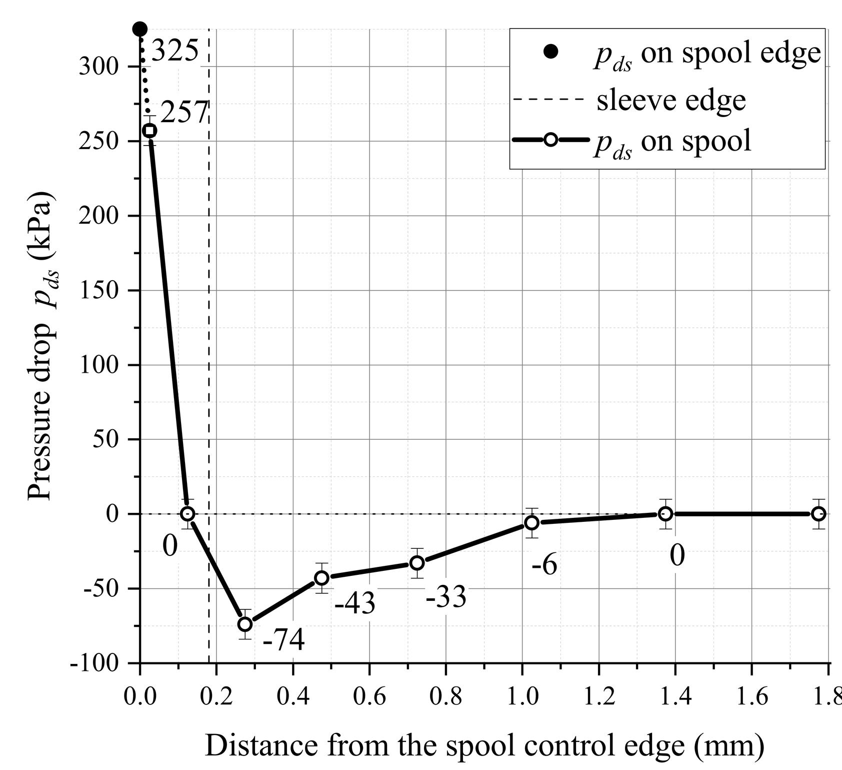
lugowskj@purdue.edu

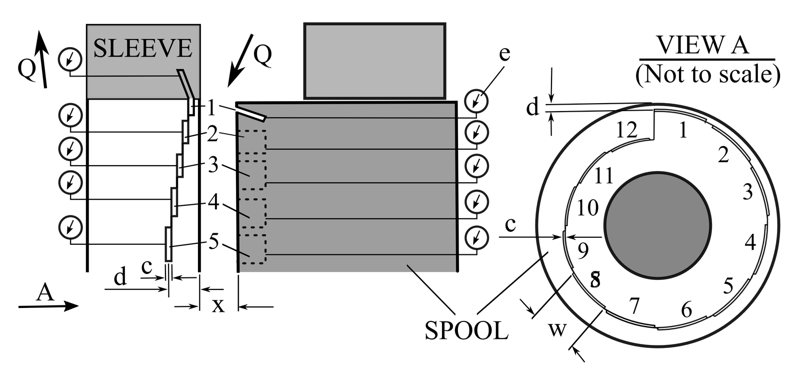


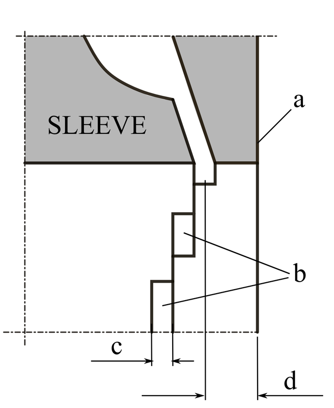




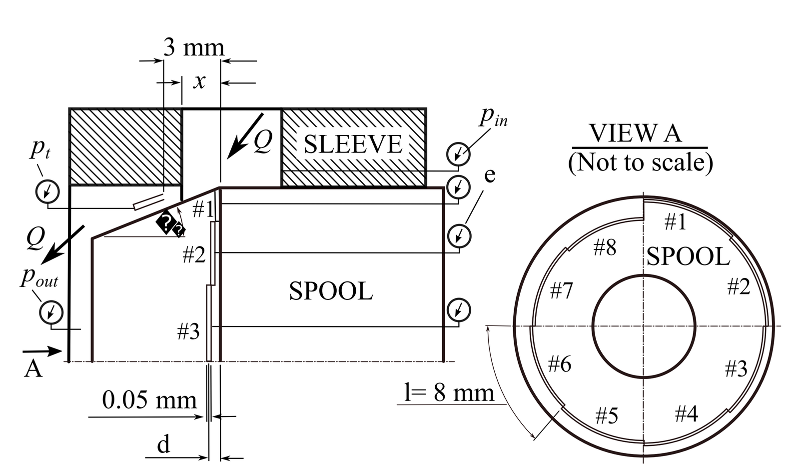




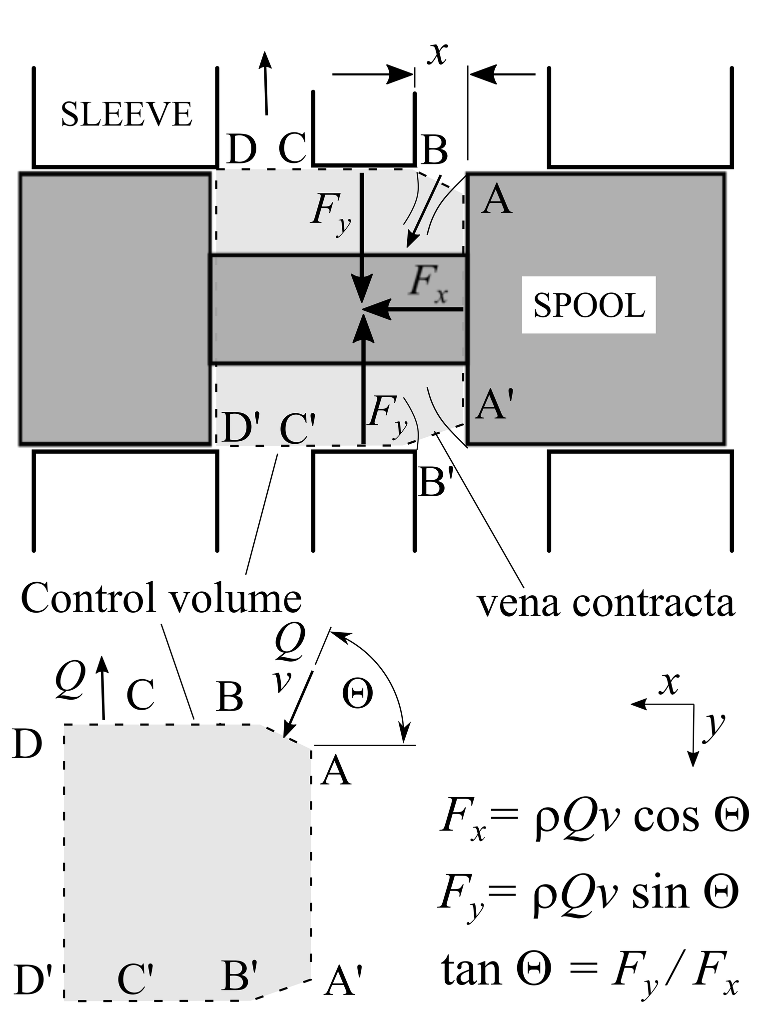




Tap #	c mm	d (spool) mm	d (sleeve) mm
1	0.04	0.10	0.12
2	0.04	0.14	0.16
3	0.04	0.18	0.20
4	0.04	0.22	0.24
5	0.06	0.26	0.29
6	0.08	0.32	0.36
7	0.10	0.40	0.45
8	0.12	0.50	0.56
9	0.14	0.62	0.69
10	0.16	0.76	0.84
11	0.18	0.92	1.01
12	0.20	1.10	1.20



Tap #	1	2	3	4	5	6	7	8
d, mm	0.025	0.125	0.275	0.475	0.725	1.025	1.375	1.775



Existence and Smoothness
of the Navier–Stokes Equation

CHARLES L. FEFFERMAN

The Euler and Navier–Stokes equations describe the motion of a fluid in $\mathbb{R}^n (n = 2 \text{ or } 3)$. These equations are to be solved for an unknown velocity vector $u(x,t) = (u_i(x,t))_{1 \le i \le n} \in \mathbb{R}^n$ and pressure $p(x,t) \in \mathbb{R}$, defined for position $x \in \mathbb{R}^n$ and time $t \ge 0$. We restrict attention here to incompressible fluids filling all of $\mathbb{R}^n$. The *Navier–Stokes* equations are then given by

$$\frac{\partial}{\partial t} u_i + \sum_{j=1}^{n} u_j \frac{\partial u_i}{\partial x_j} = \nu \Delta u_i - \frac{\partial p}{\partial x_i} + f_i(x,t) \qquad (x \in \mathbb{R}^n, t \ge 0), \tag{1}$$

$$\operatorname{div} u = \sum_{i=1}^{n} \frac{\partial u_i}{\partial x_i} = 0 \qquad (x \in \mathbb{R}^n, t \ge 0) \tag{2}$$

with initial conditions

$$u(x,0) = u^\circ(x) \qquad (x \in \mathbb{R}^n). \tag{3}$$

ABSTRACT

We explain the origin of the flow force in a hydraulic spool valve by the static-pressure distribution in the valve orifice. Our pressure measurements in the uncompensated valve point to the jet attachment to a wall and the resulting imbalance of the static pressure that produces the flow force. The jet attachment, known as the Coanda effect, takes place along a short distance, less than one millimeter at the flow parameters we used, and is often accompanied by a pressure buildup that most likely causes the jet ejection away from the wall. Another experiment with a beveled spool revealed high pressure acting on an area of the spool upstream from the valve orifice that produces the compensating flow force. The measurements of the dynamic pressure three millimeters downstream from the orifice point to an almost complete loss of kinetic energy in the jet, and thus the upstream area on the spool is the likely only place where the compensation of the flow force takes place. Based on those findings, we present modified formulas for the flow forces.

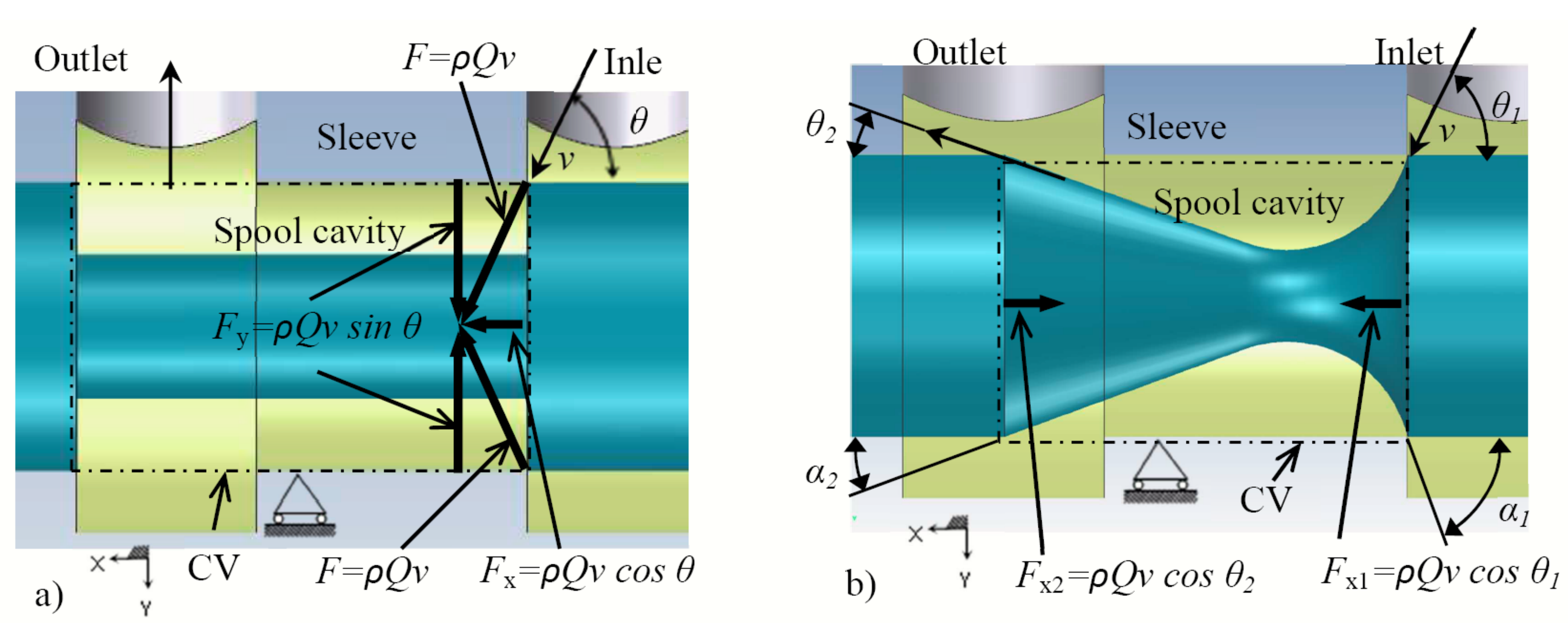


Fig. 2: *Cross-section of a valve with an uncompensated valve, where fluid enters the control volume (CV) at maximum velocity v and slows down to zero velocity inside of CV generating axial flow force F_x acting on the spool to close the valve (a) and the compensated spool with a turbine-bucket profile along which the jet maintains its maximum velocity v, generating negative compensating flow force F_{x2} as it leaves CV (b)*

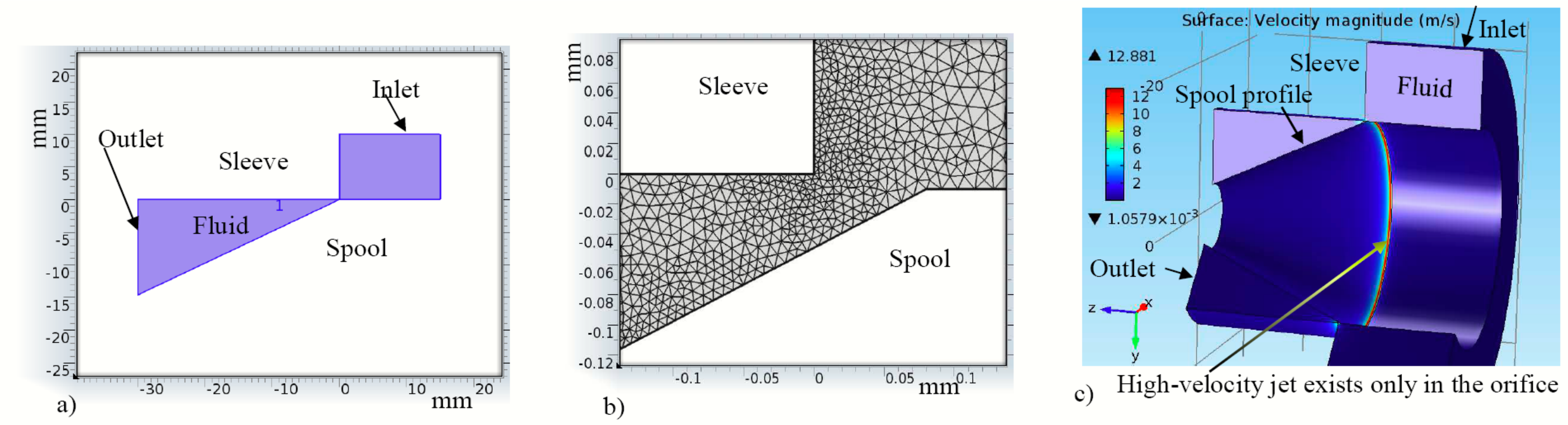


Fig. 9: *2-D geometry used for finite element model in COMSOL (a), the free triangular mesh shown for orifice area (b), and the velocity magnitude field for the 2-D axisymmetric geometry used for the comparison of computations (c)*

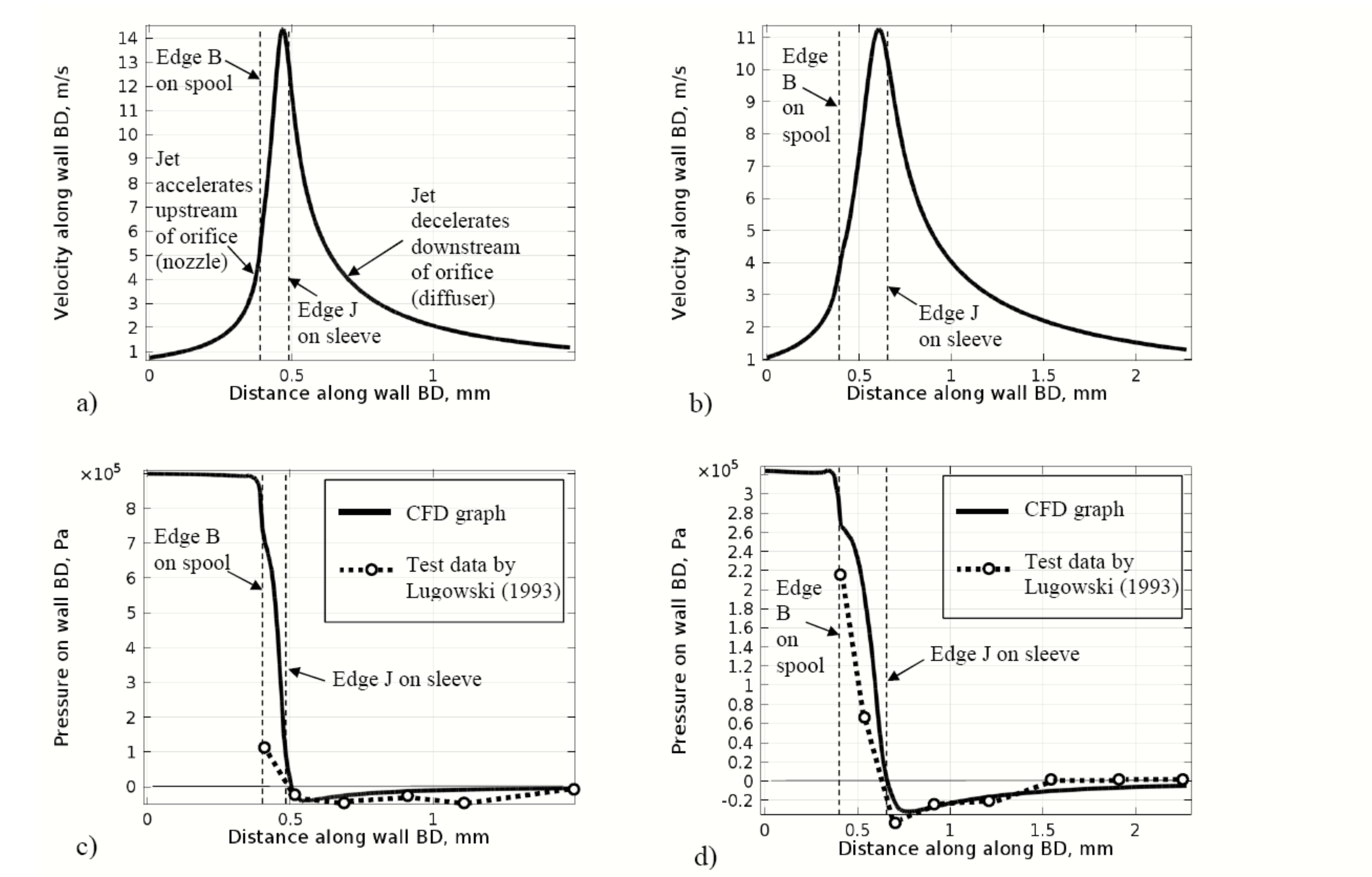


Fig. 11: *Final-element calculations for the velocity of fluid and the pressure along spool wall BD, see also Figs. 4 and 10, with the cutline in COMSOL model extended by 0.4 mm beyond the spool control edge (point B); All corresponding parameters as in Fig. 10: Fluid velocity at x=0.08 mm (a), and at x=0.23 mm (b), pressure on wall BD of spool at x= 0.08 mm (c), and at x= 0.23 mm (d)*