

# A Concurrent Parallel Multiscale Algorithm for Large 3D Continuum/Atomistic Simulations at Finite Temperature Using LAMMPS

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# Outline

- Background
- Implementations
- Applications to Dislocations in Aluminum
- Summary

The “Need for (Computational) Speed...”



# ...A Beautiful Example of Failure



# Failure at different length-scales

Characteristic  
Time-scale ↑

Engineering

Materials  
Characterization

Atomistic  
Studies

Quantum  
Theory

orbital structure

H atom  
pm

nm

μm

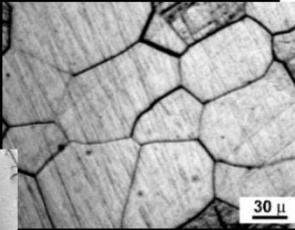
mm

m

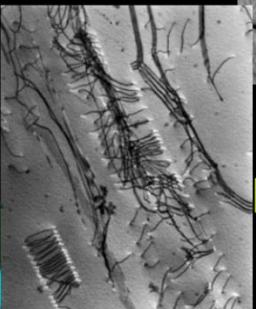
Characteristic  
Length-scale



structure



component

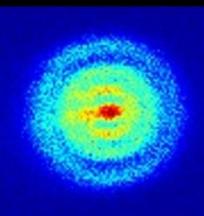


grain boundaries in  
polycrystalline metal

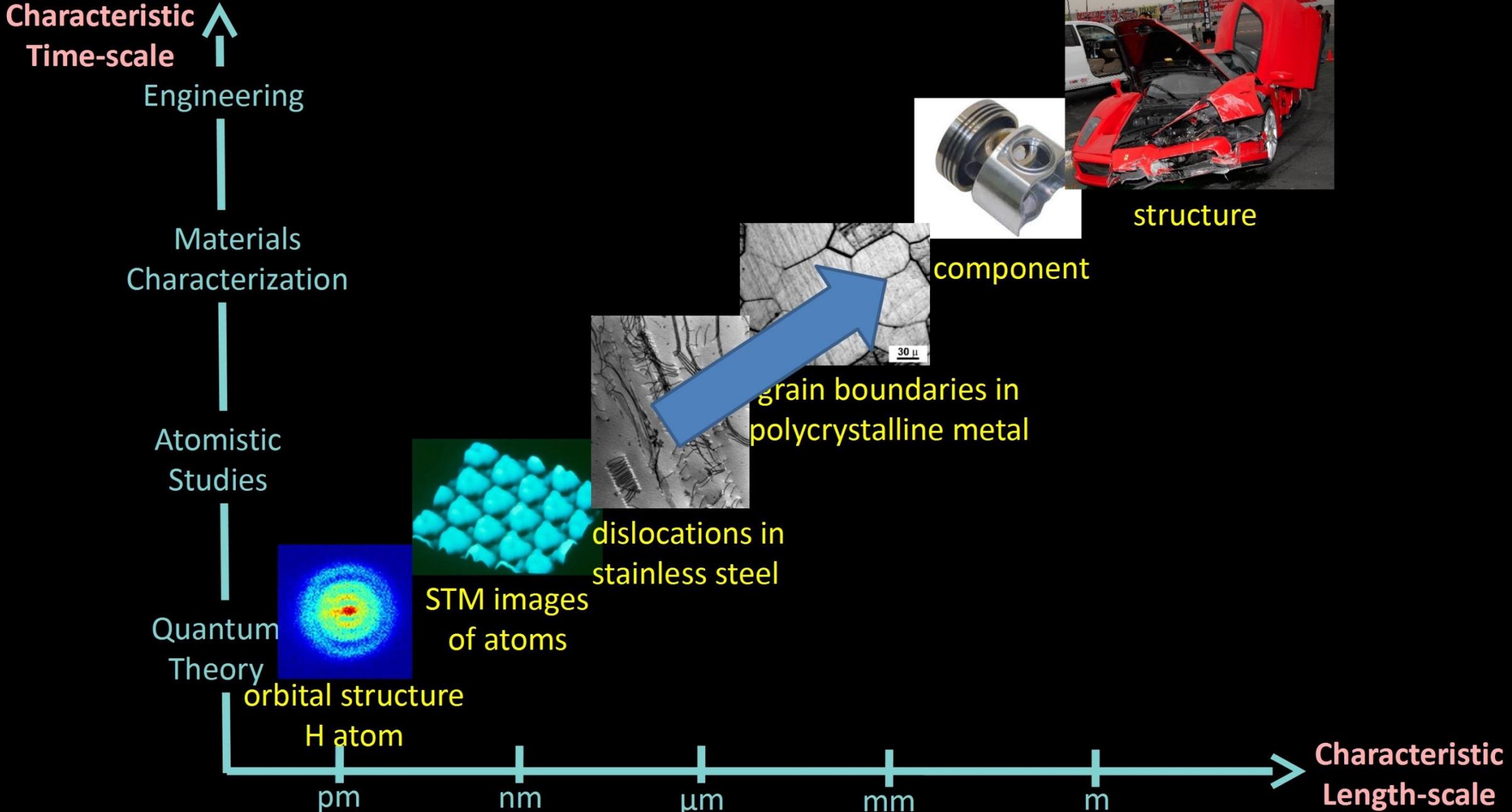


STM images  
of atoms

dislocations in  
stainless steel



# Failure at different length-scales



# Failure at different length-scales

FEM / finite differences / multibody simulations

Characteristic  
Time-scale ↑

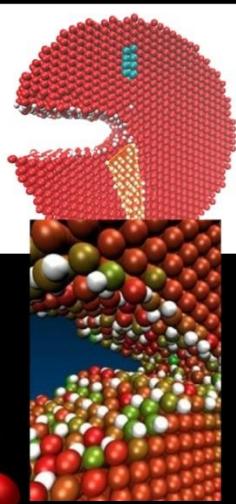
Engineering

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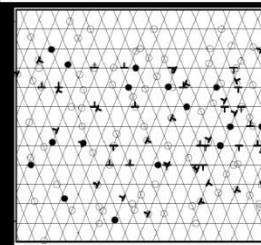
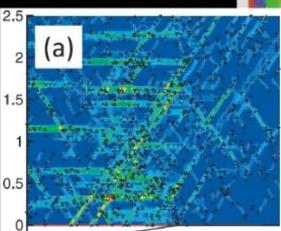
ab-initio  
electronic  
structure



atoms

discrete  
dislocations

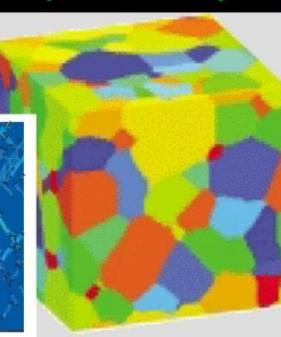
molecular  
dynamics



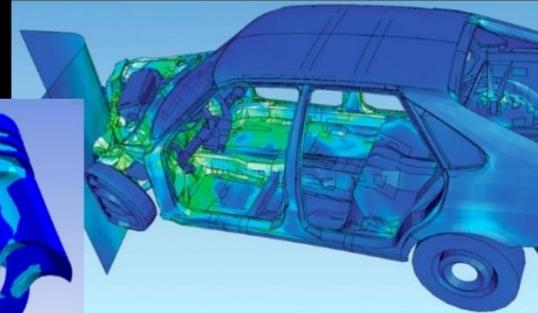
polycrystalline  
plasticity

polycrystalline  
material

dislocations



component



structure

Characteristic  
Length-scale →

pm

nm

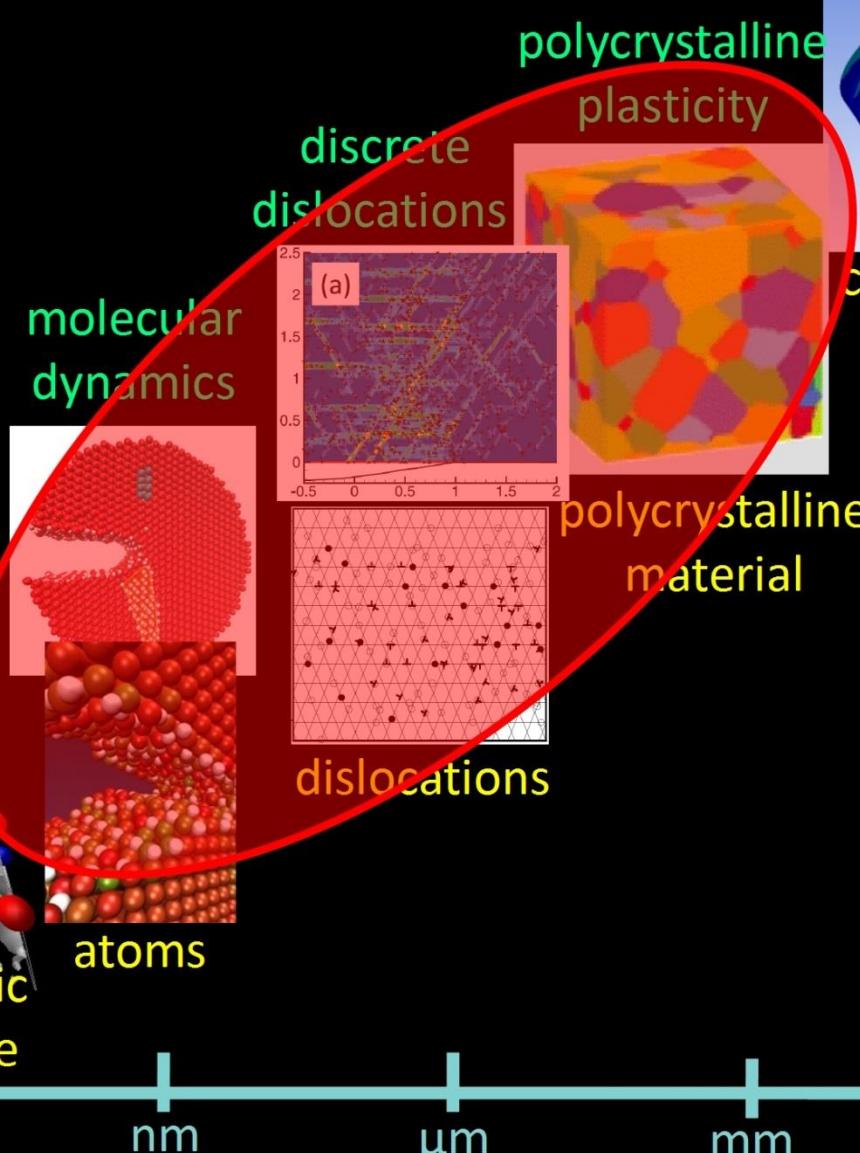
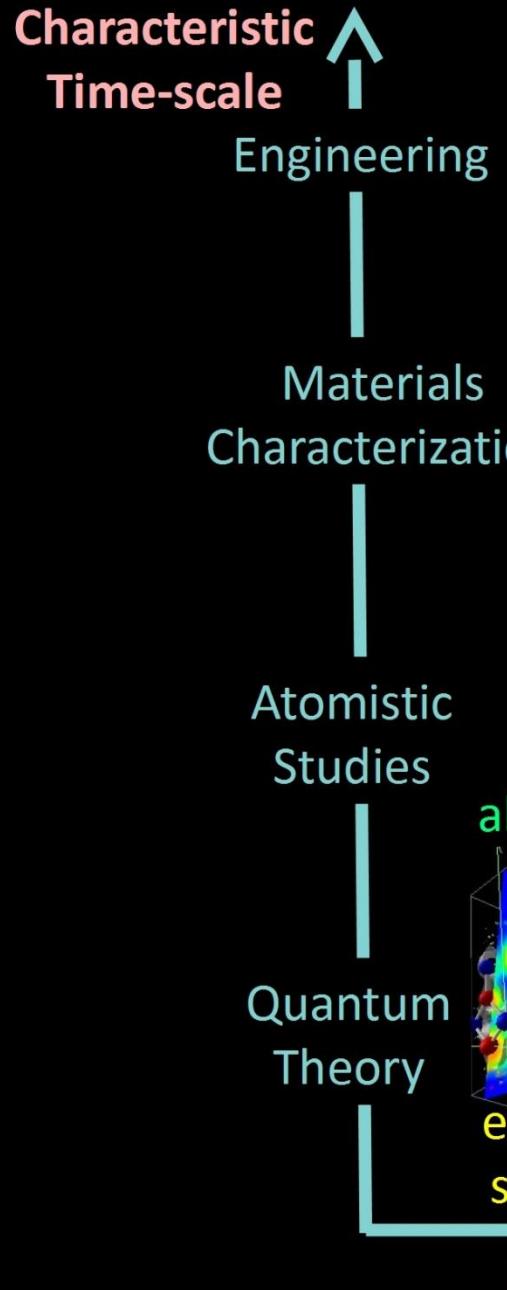
μm

mm

m

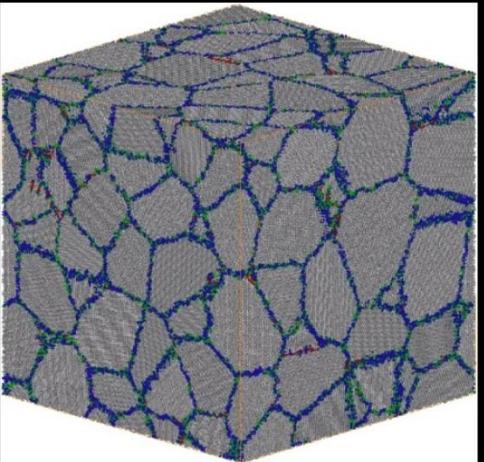
# Failure at different length-scales

FEM / finite differences / multibody simulations

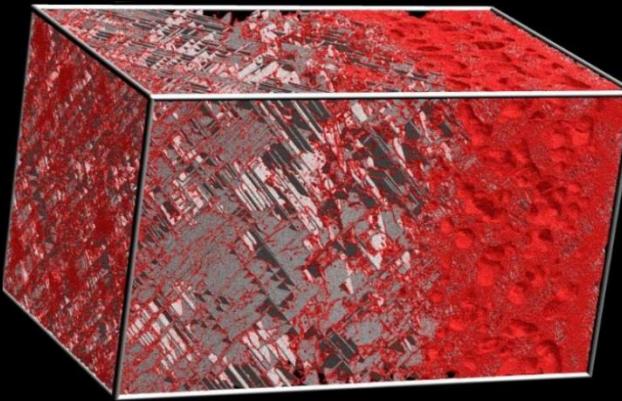


# Possible Strategies

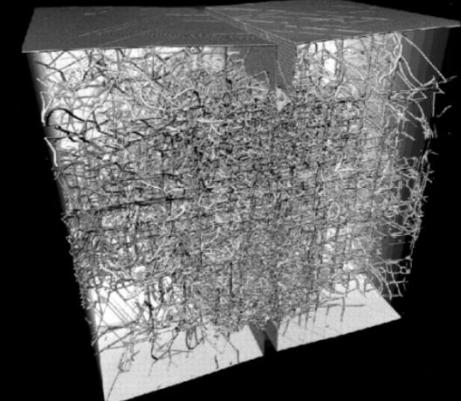
## (1) Very large atomic models



Van Swygenhoven H., Weertman J. R.,  
Materials Today 2006



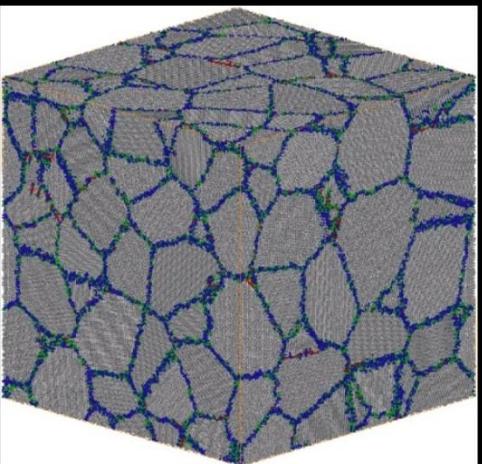
Germann T. C., Kadauy K., Lomdahl P. S.,  
Gordon Bell Performance Prize 2005



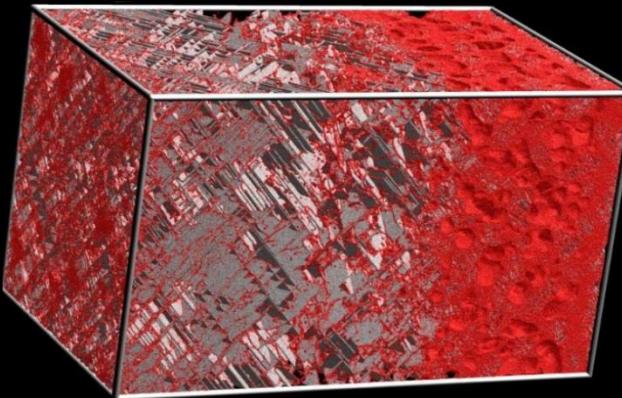
Abraham et al.,  
PNAS 2001

# Possible Strategies

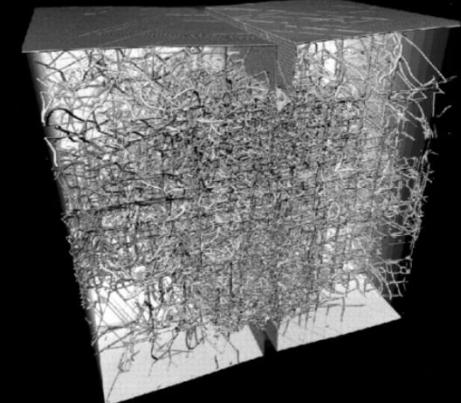
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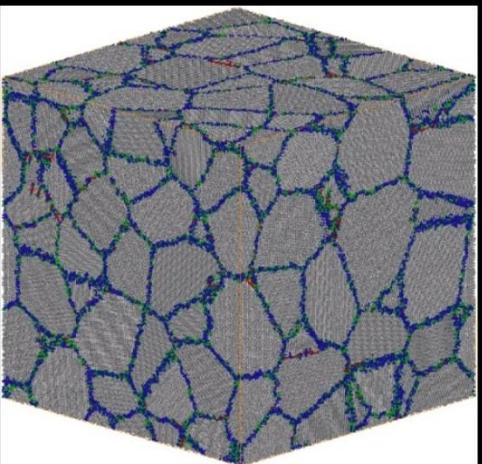
## (2) Hierarchical / Sequential Multiscale Models



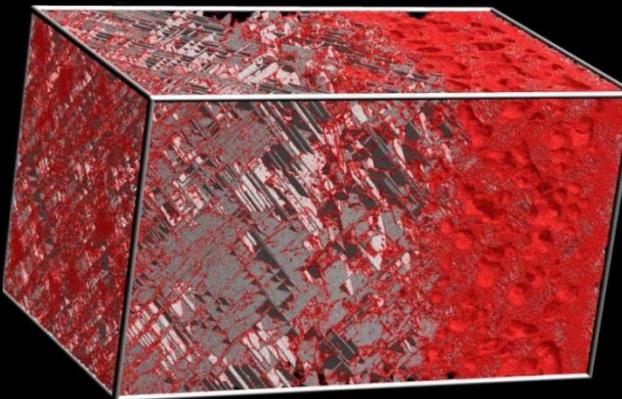
**weak coupling  
between scales**

# Possible Strategies

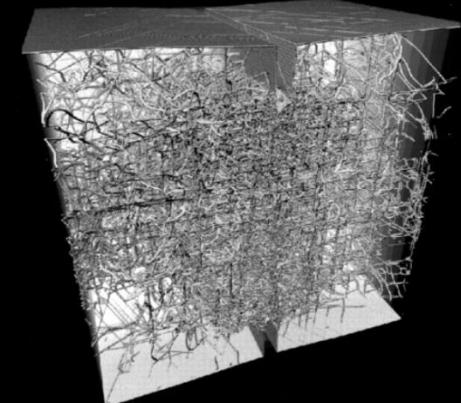
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Van Swygenhoven H., Weertman J. R.,  
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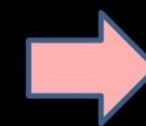


Germann T. C., Kadauy K., Lomdahl P. S.,  
Gordon Bell Performance Prize 2005



Abraham et al.,  
PNAS 2001

## (2) Hierarchical / Sequential Multiscale Models



**weak coupling  
between scales**

## (3) Concurrent Multiscale Models

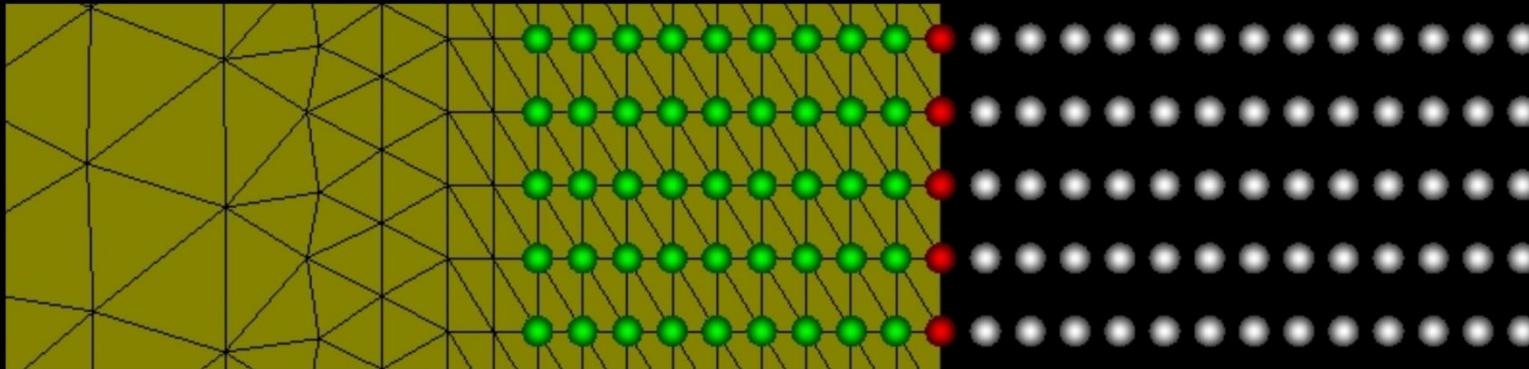


**strong coupling  
between scales**

# Our Multiscale Approach

**CADD-like Coupling** (by Shilkrot, Miller and Curtin 2002, *PRL* 89(2) #025501) **in 3D**

pad (P)



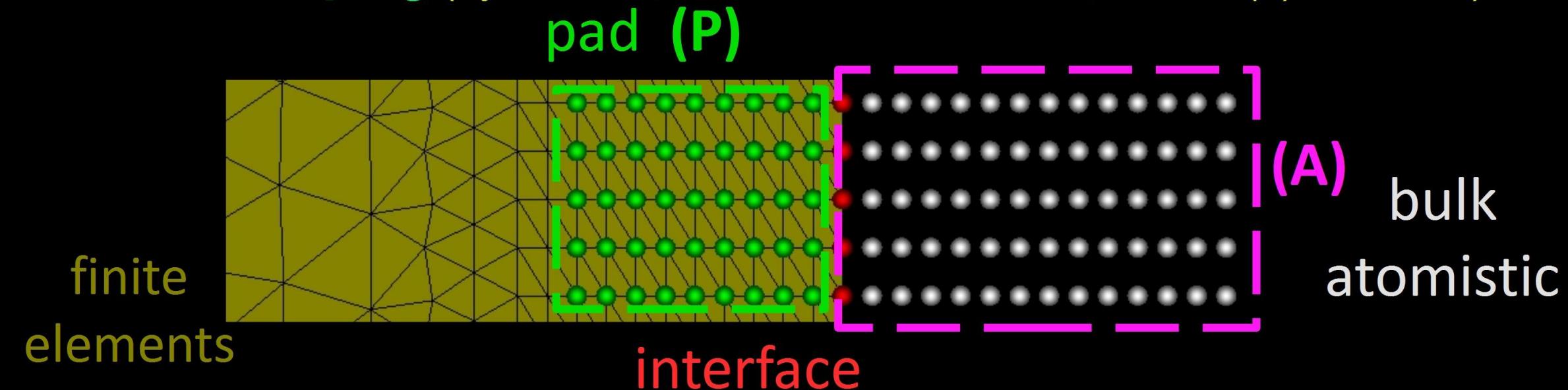
finite  
elements

interface

bulk  
atomistic

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**CADD-like Coupling** (by Shilkrot, Miller and Curtin 2002, *PRL* 89(2) #025501) **in 3D**

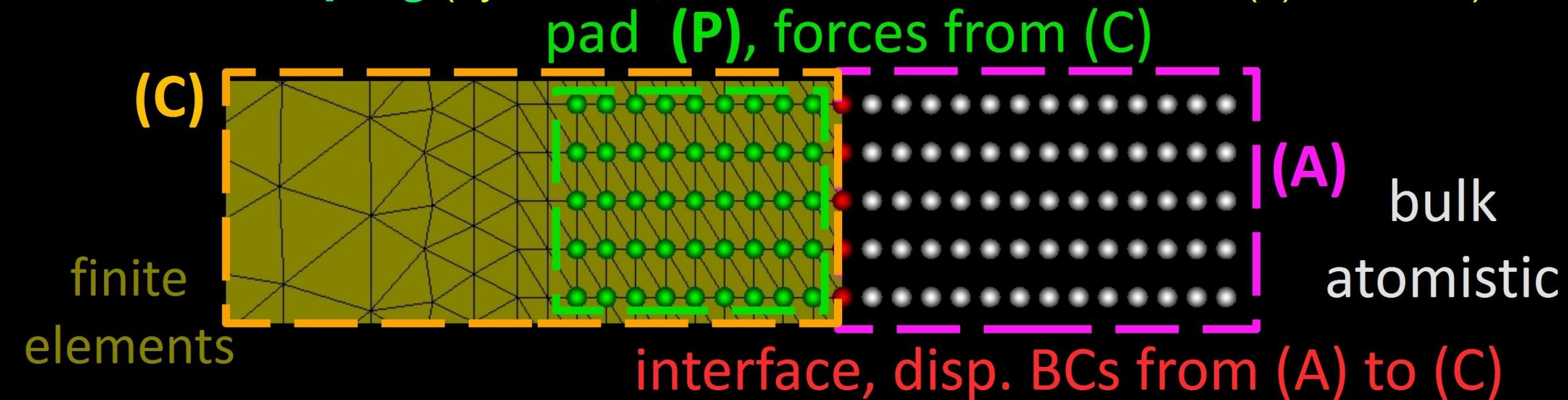


**Atoms**      Energy      Forces

$$W^{A+P} = \sum_{i \in \Omega^A, \Omega^P} E^i \quad \rightarrow \quad \mathbf{F}^i = -\frac{\partial}{\partial \mathbf{r}^i} (W^{A+P}) \quad i \in \Omega^A$$

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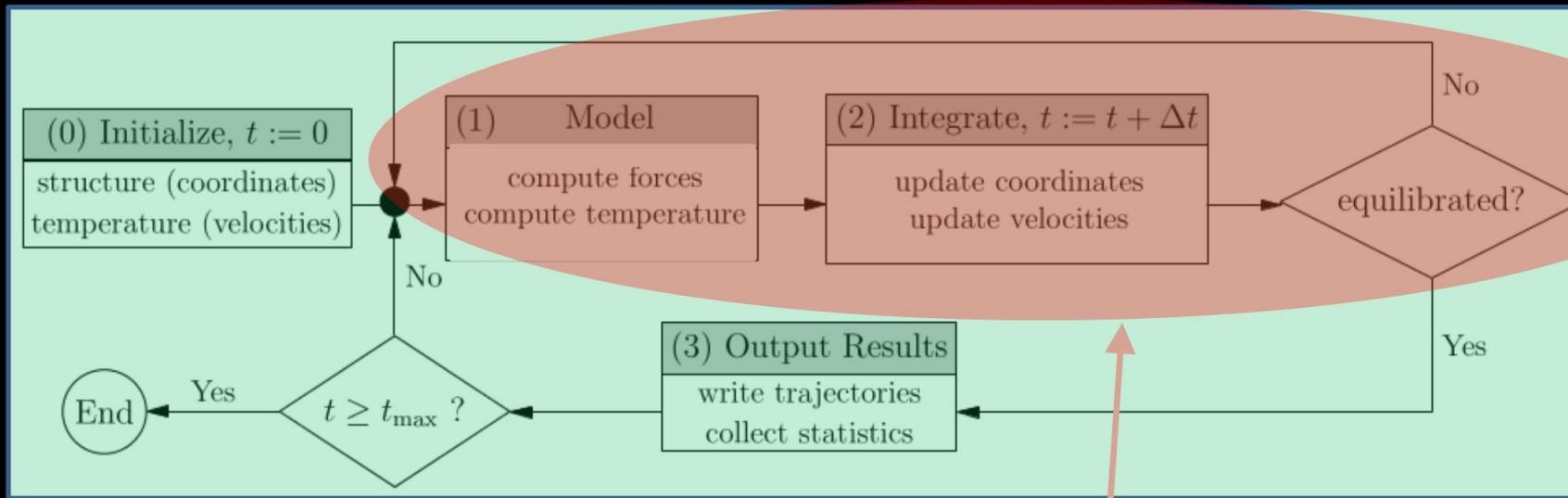


**Atoms** | 
$$W^{A+P} = \sum_{i \in \Omega^A, \Omega^P} E^i \rightarrow \mathbf{F}^i = -\frac{\partial}{\partial \mathbf{r}^i} (W^{A+P}) \quad i \in \Omega^A$$

**Nodes** | 
$$W^{C+P} = \int_{\Omega^{C+P}} W(\mathbf{r}) d\Omega$$
  
$$W(\mathbf{r}) = \frac{1}{2} \epsilon_{ij}(\mathbf{r}) \epsilon_{kl}(\mathbf{r}) C_{ijkl}$$
  
$$\rightarrow \mathbf{F}^i = -\frac{\partial}{\partial \mathbf{r}^i} (W^{C+P}) \quad i \in \Omega^{C+P} \rightarrow \mathbf{F} = \sum_{e=1}^{n_{elem}} \int_{V_e} \mathbb{B}^T \mathbb{D} \mathbb{B} \mathbf{U}_e dV$$

# Our Multiscale Approach

Explicit Dynamics (**Velocity Verlet** + **Langevin**) for both MD and FE regions

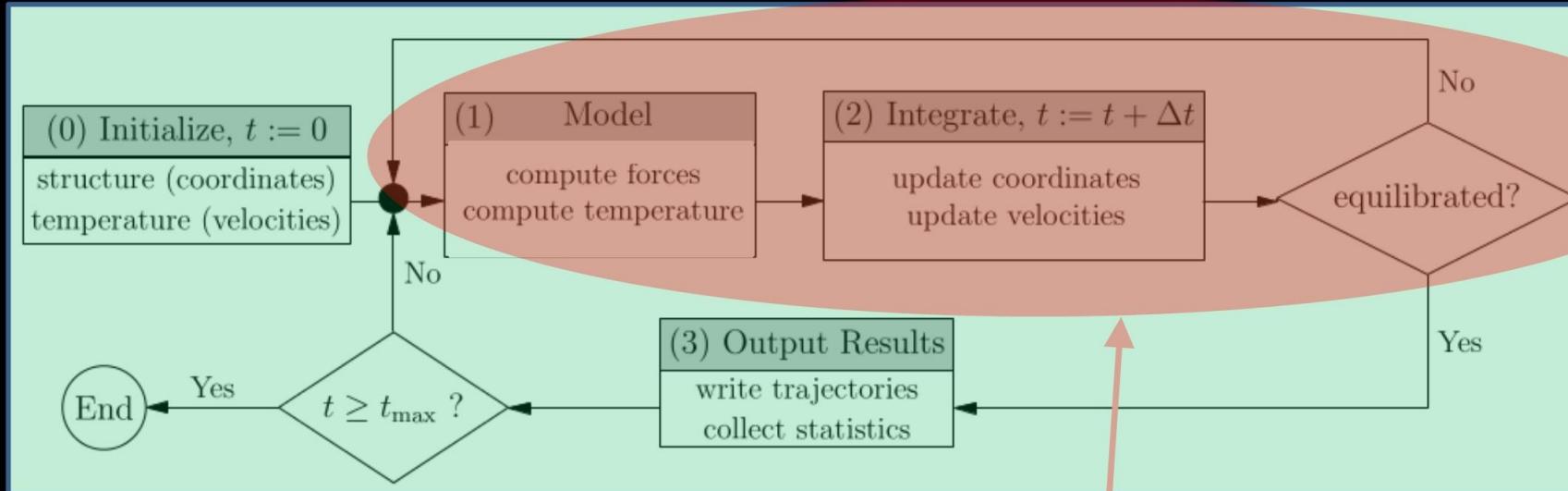


```
while t ≤ tfinal do
     $r_{t+Δt} := r_t + v_t Δt + \hat{f}_t \frac{(\Delta t)^2}{2m}$ 
     $v_{t+Δt/2} := v_t + \frac{\hat{f}_t}{2m} Δt$ 
     $\hat{f}_{t+Δt} := f^L(r_{t+Δt}, v_{t+Δt/2})$ 
     $v_{t+Δt} := v_{t+Δt/2} + \frac{\hat{f}_{t+Δt}}{2m} Δt$ 
    t := t + Δt
end while
```

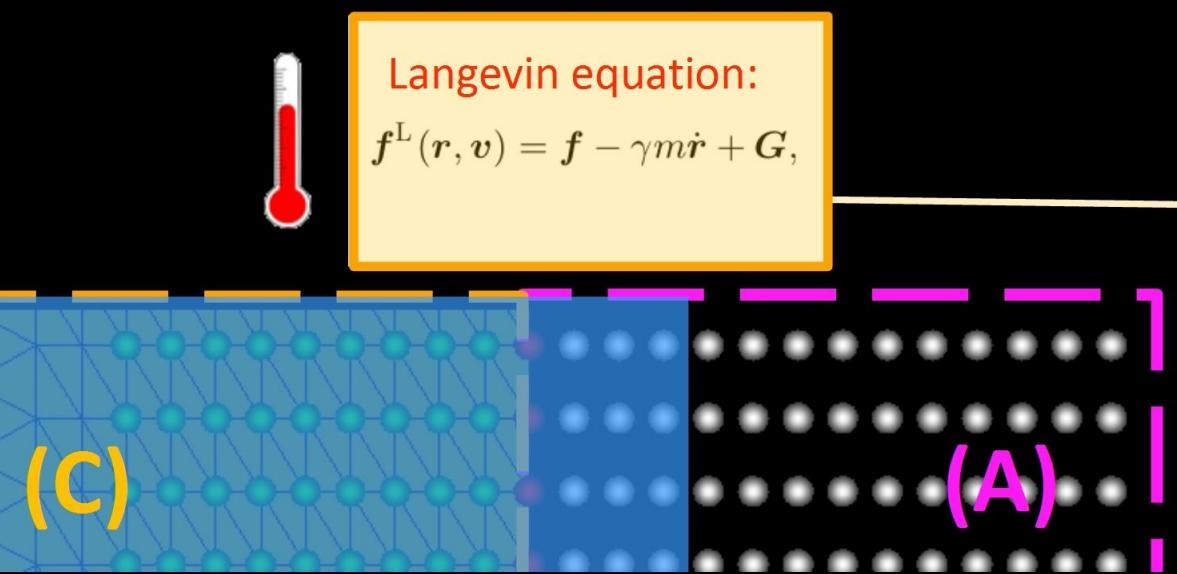
- ← positions end of step
- ← velocities half of step
- ← forces end of step
- ← velocities end of step

# Our Multiscale Approach

Explicit Dynamics (**Velocity Verlet + Langevin**) for both MD and FE regions



Absorb reflected waves from atomistic region



Langevin equation:

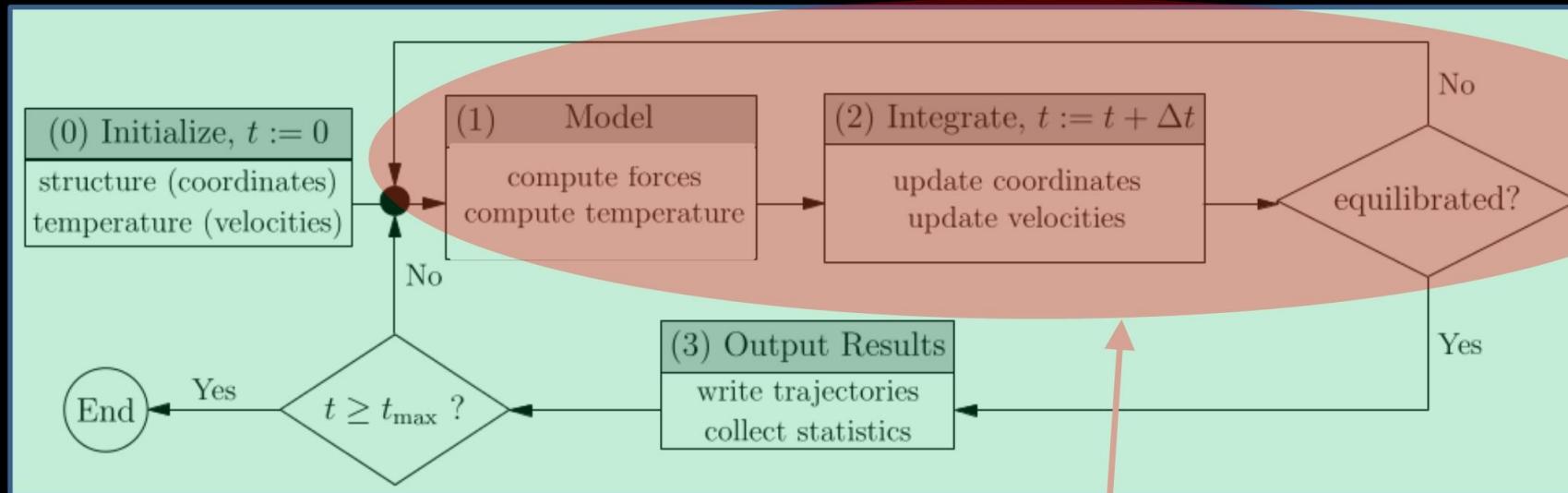
$$f^L(r, v) = f - \gamma m \dot{r} + G,$$

```
while t ≤ t_final do
    r_{t+Δt} := r_t + v_t Δt +  $\hat{f}_t \frac{(\Delta t)^2}{2m}$ 
    v_{t+Δt/2} := v_t +  $\frac{\hat{f}_t}{2m} \Delta t$ 
     $\hat{f}_{t+Δt} := f^L(r_{t+Δt}, v_{t+Δt/2})$ 
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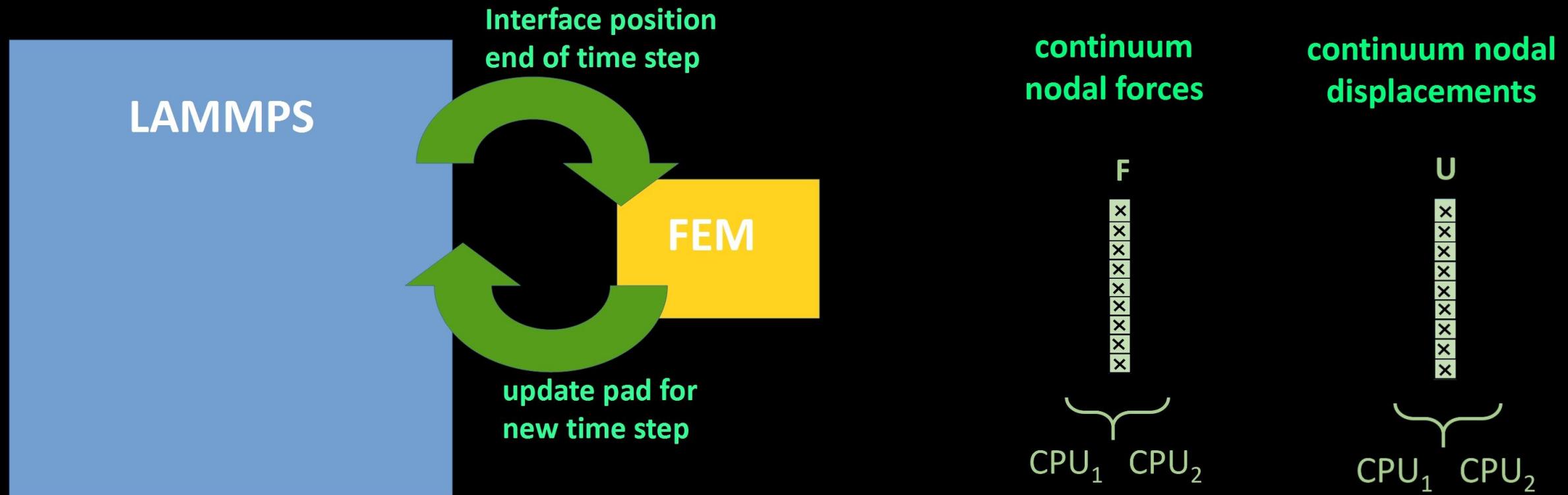
```
while t ≤ tfinal do
    rt+Δt := rt + vtΔt +  $\hat{f}_t \frac{(\Delta t)^2}{2m}$ 
    vt+Δt/2 := vt +  $\frac{\hat{f}_t}{2m} \Delta t$ 
     $\hat{f}_{t+Δt}$  := fL(rt+Δt, vt+Δt/2)
    vt+Δt := vt+Δt/2 +  $\frac{\hat{f}_{t+Δt}}{2m} \Delta t$ 
    t := t + Δt
end while
```

- ← positions end of step
- ← velocities half of step
- ← forces end of step
- ← velocities end of step

Use LAMMPS for the parallel coupling with the forces on pad from FE

# Our Multiscale Approach - LAMMPS

LAMMPS as main driver code, calling the FEM subroutines (C++) when needed

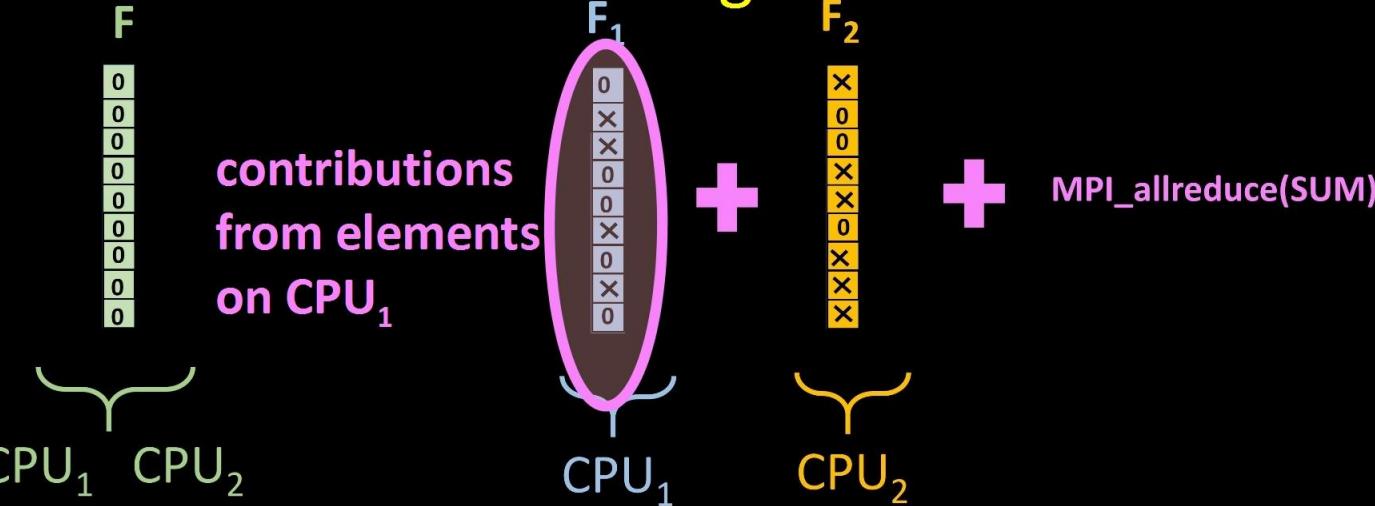


Additional FE dofs (nodal values) stored in global arrays, while tables of finite elements distributed among the CPUs

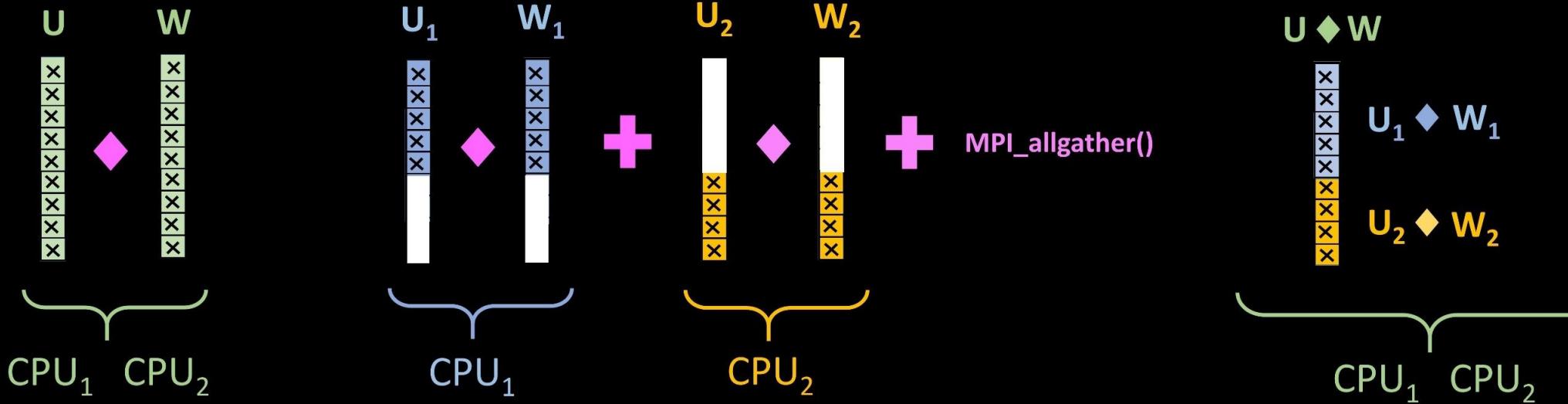
# Our Multiscale Approach - Parallelization

Operations on FE dofs distributed among different CPUs used for LAMMPS

deformation forces in continuous region:

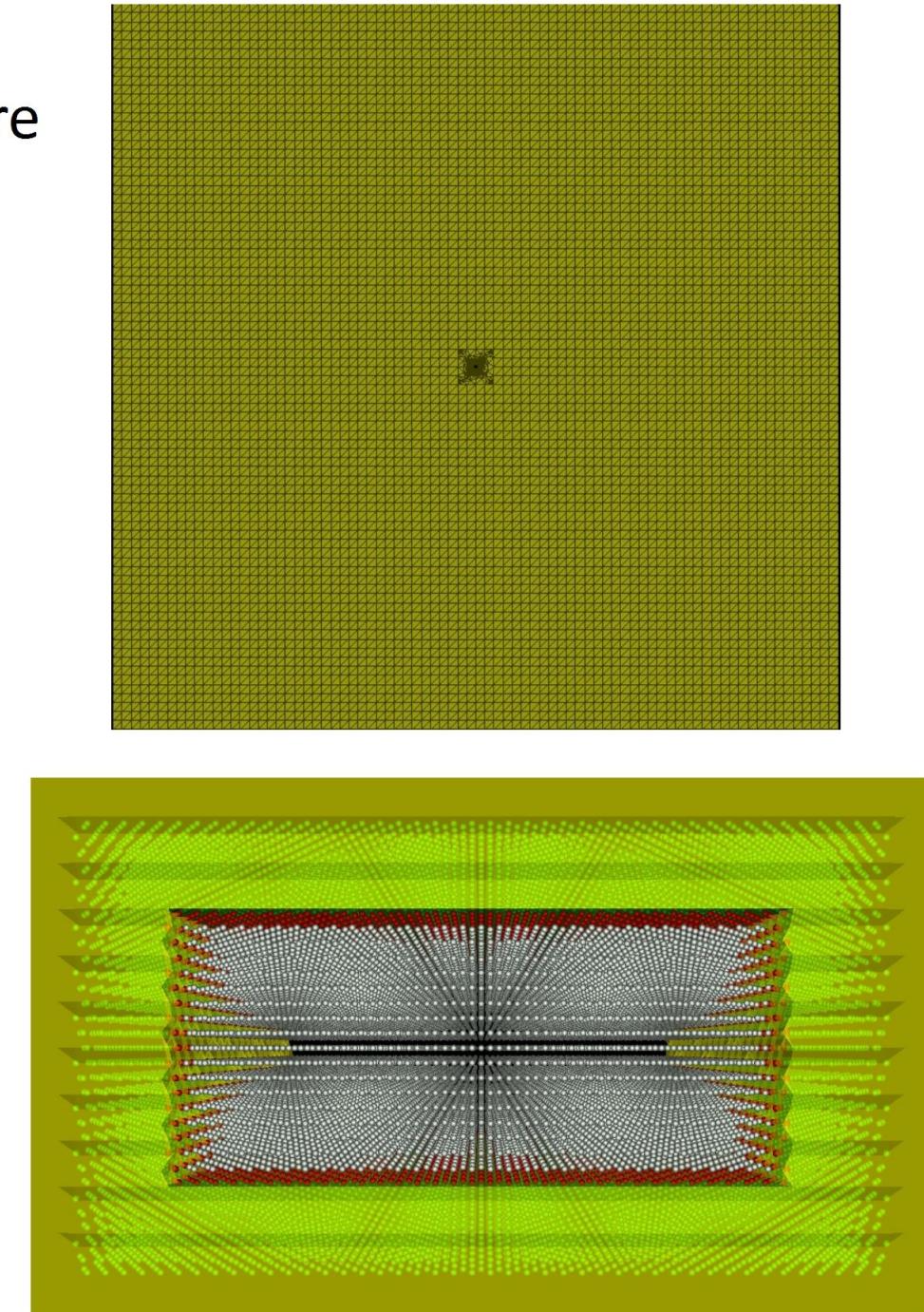
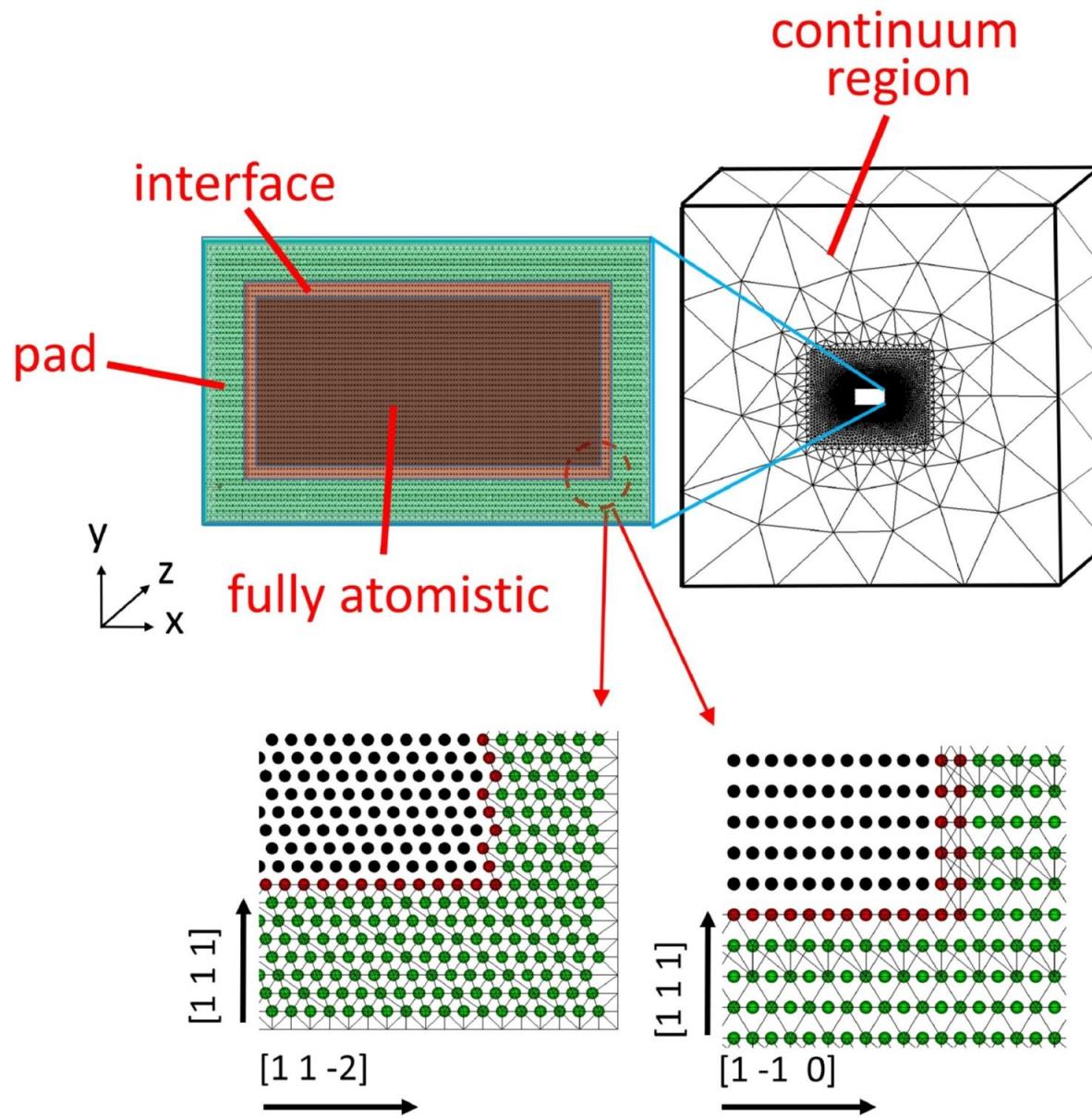


vector-vector operations:



# A Multiscale Thin Plate

Modeling a block of aluminum at finite temperature



# A Multiscale Thin Plate - Benchmark

## MODEL A (big)

### Continuum:

4000x4000x52nm

2864134 nodes

17170903 elements

### Atomistic:

16x9x52nm

458850 atoms

239400 pad-atoms

**9968952 DOF**  
**128 CPUs**

## MODEL B (small)

4000x4000x11nm

278337 nodes

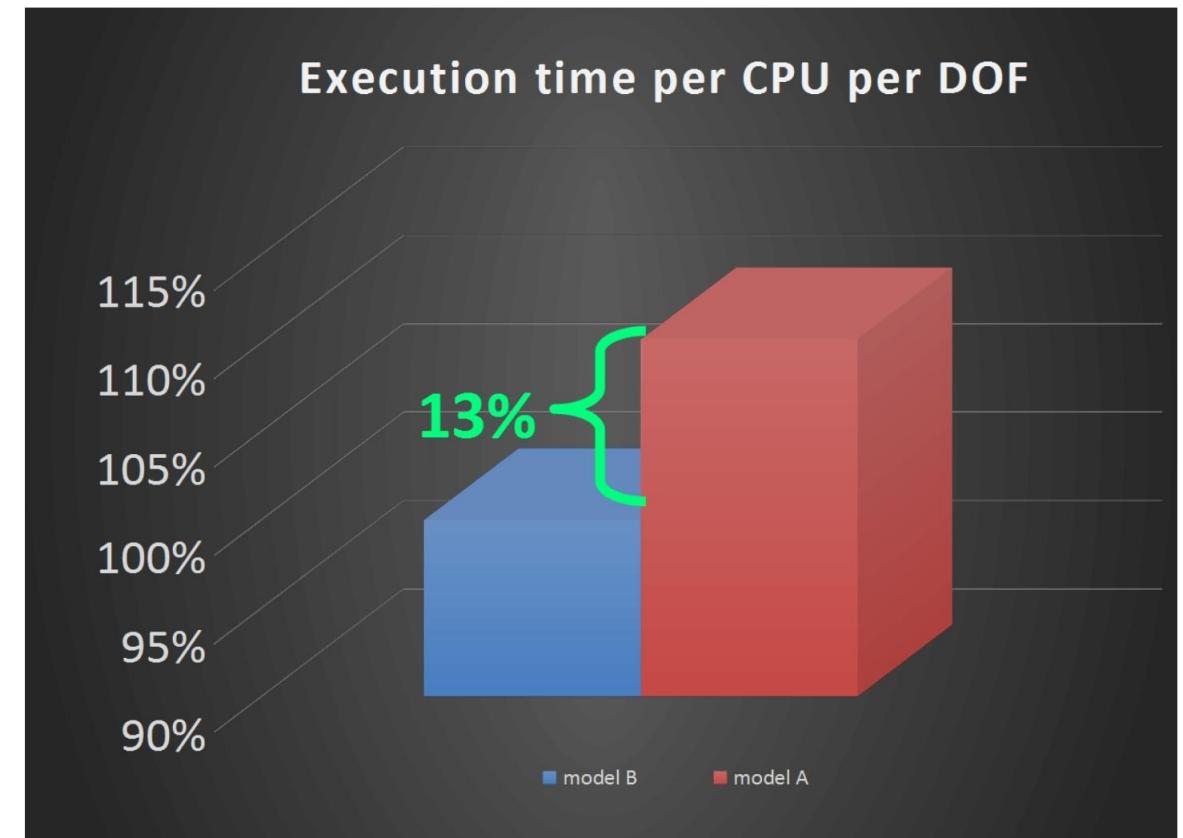
1575678 elements

9x4x11nm

55170 atoms

29349 pad-at

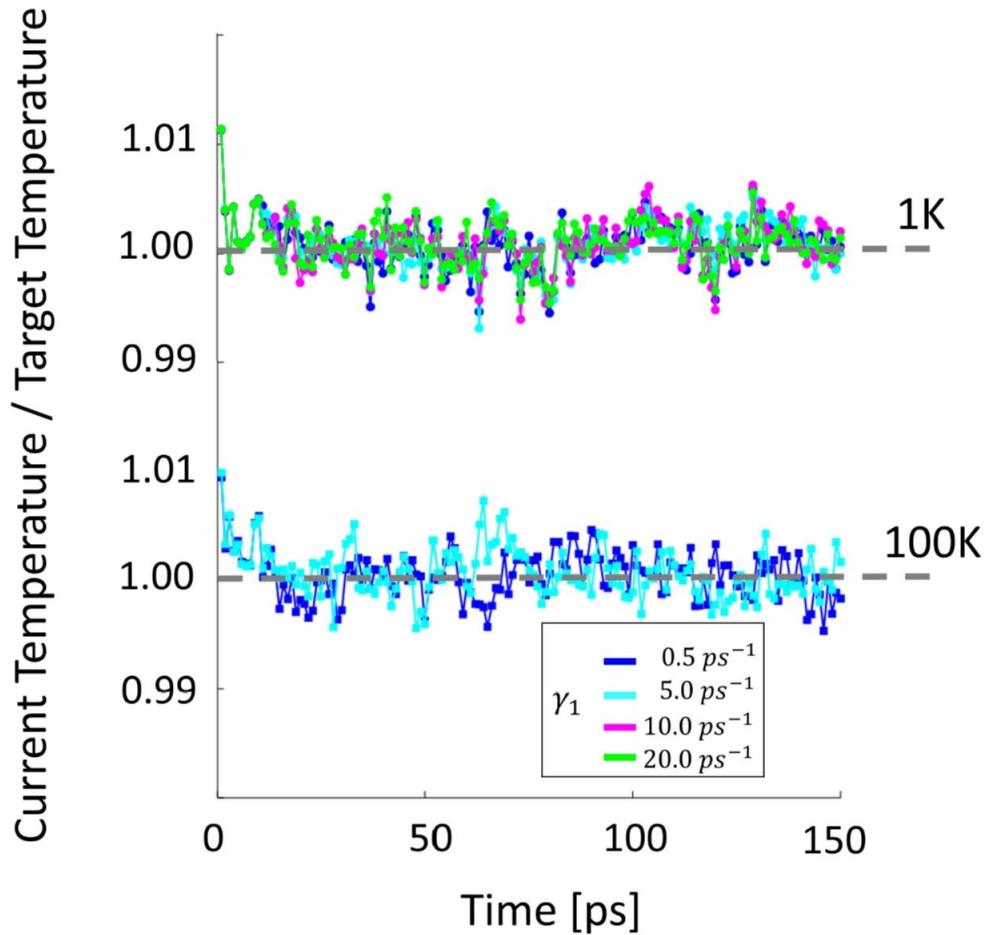
**1000521 DOF**  
**64 CPUs**



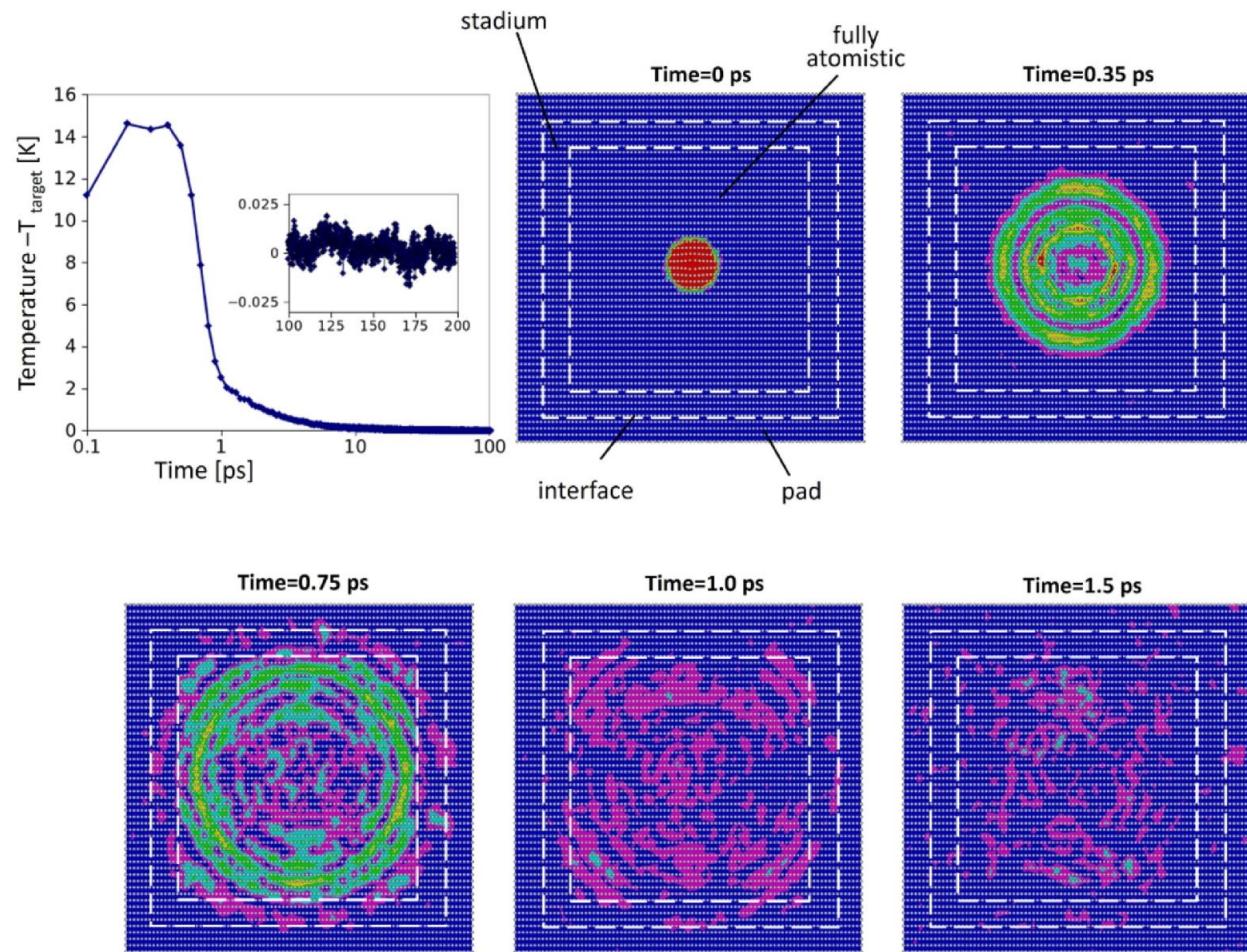
*This is how the code scales....*

# A Multiscale Thin Plate – Temperature Control

Different damping values for the continuum region with fixed damping in the atomistic



Transient Test – Pulse of Radial Displacement

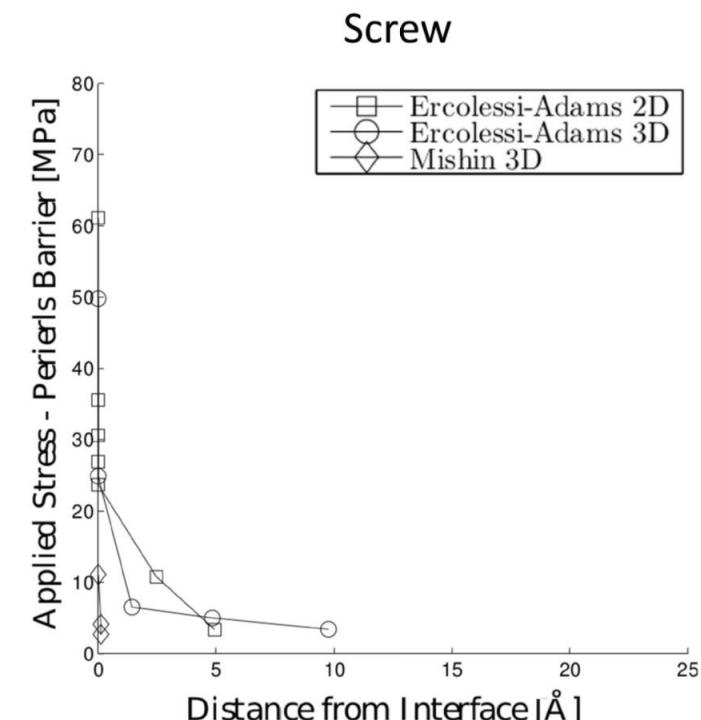
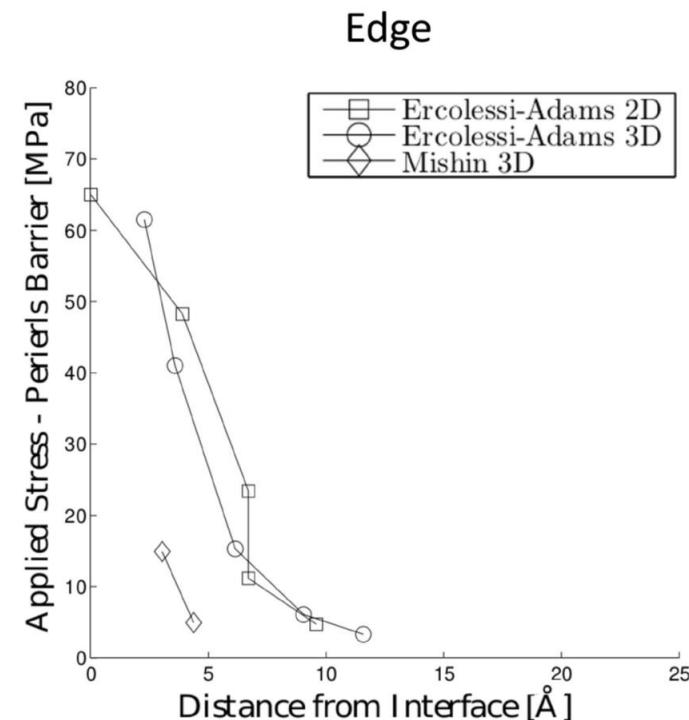
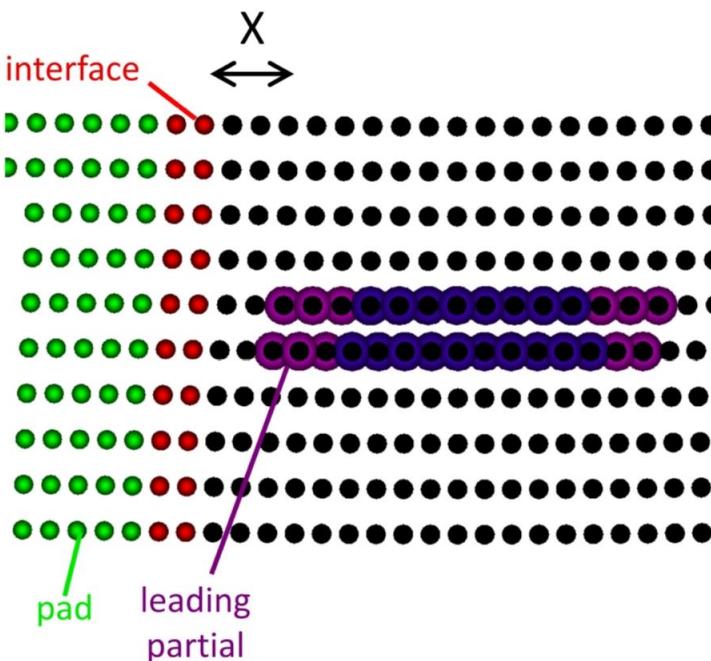


*Testing the thermostat....*

# A Multiscale Thin Plate – Insert a Dislocation and Apply Shear

Insert a dislocation (a defect) in the atomistic region and apply uniform shear strain

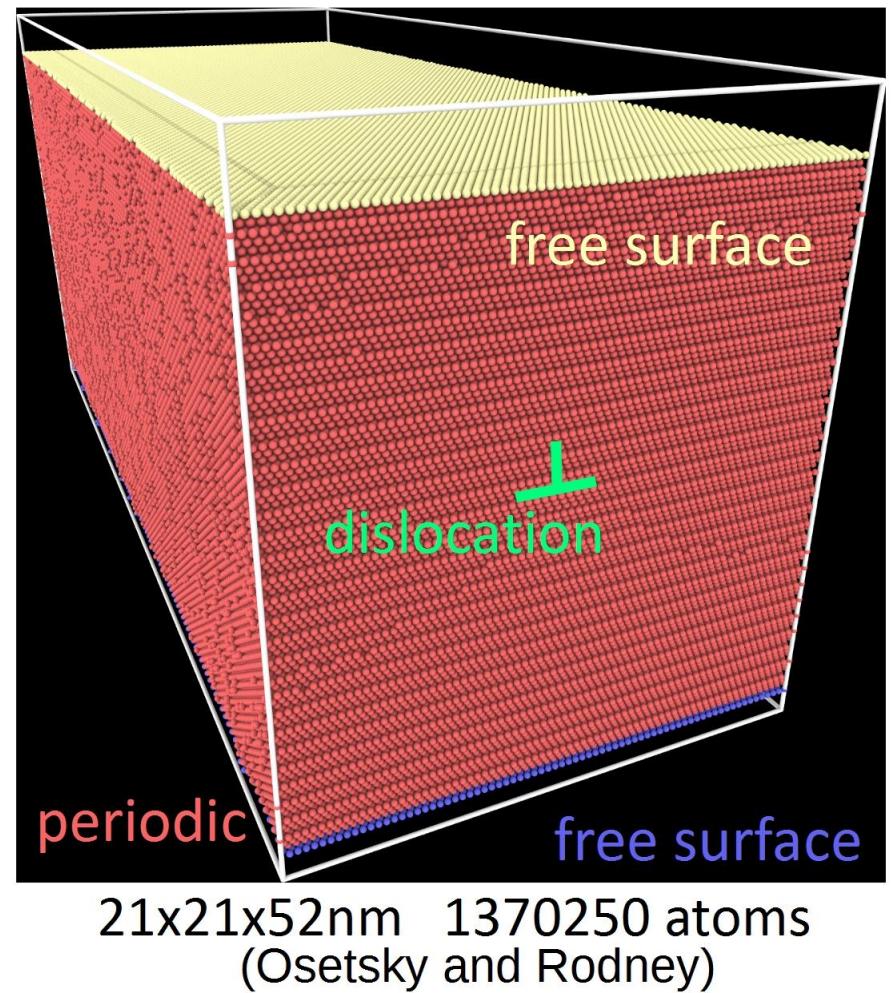
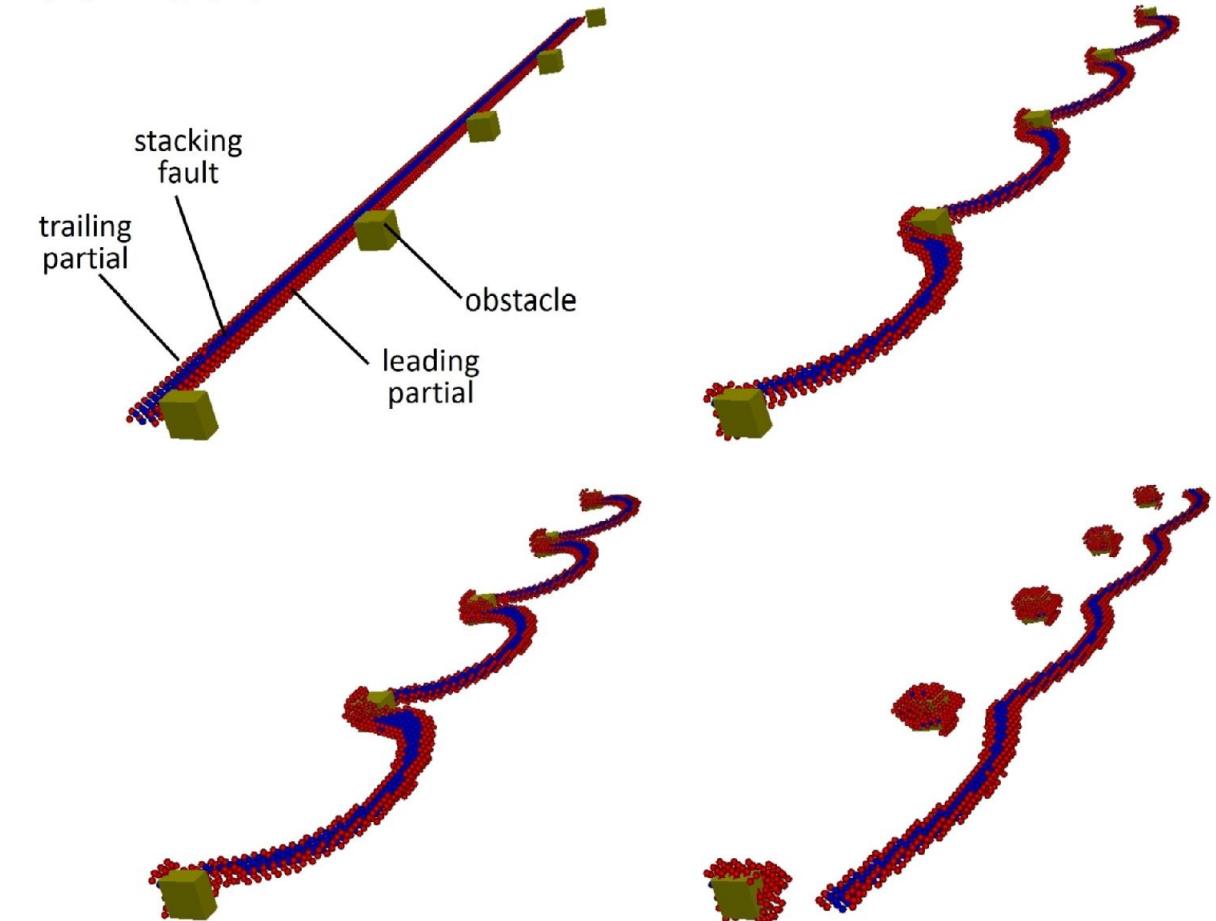
Continuum discontinuities in the displacements fields treated analytically



Spurious forces stops the dislocation **before** the interface with continuum region, high deformation at the dislocation core

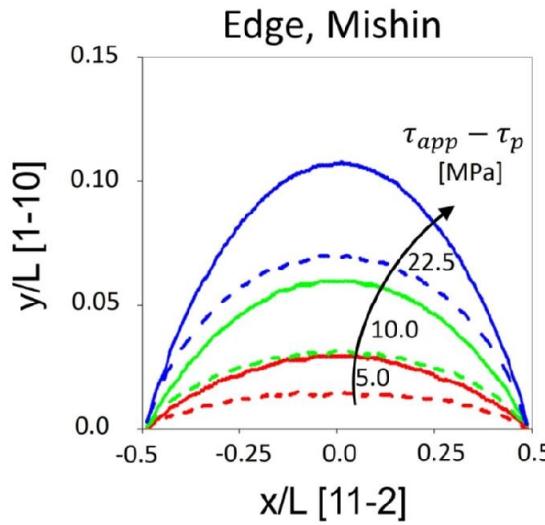
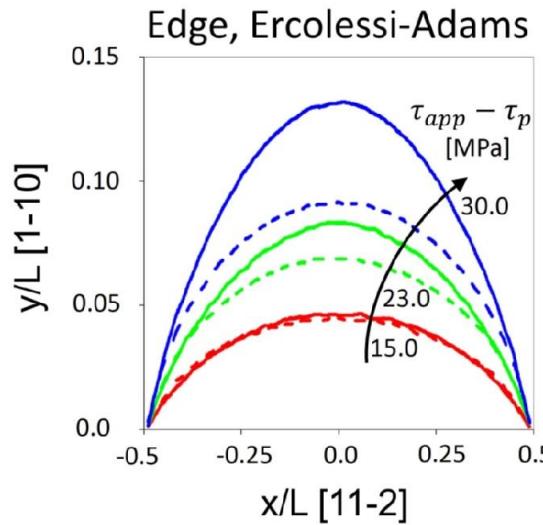
# A Multiscale Thin Plate – Disl. Bowout and Finite Size Effects

Create a field of rigid obstacles in the atomistic region to make the dislocation bowout



Deformed shapes from multiscale simulations compared to smaller finite size only atomistic results (full atomistic box of  $4000 \times 4000 \times 52 \text{ nm}$  requires 62 billions atoms!)

# A Multiscale Thin Plate – Disl. Bowout and Finite Size Effects



Infinite vs. finite size with image forces (free surfaces and periodic images)

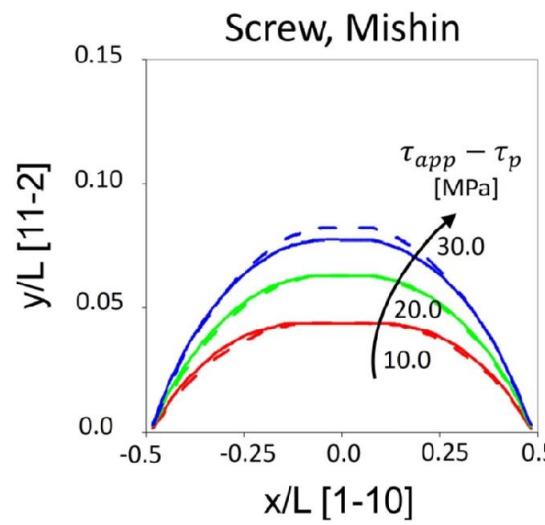
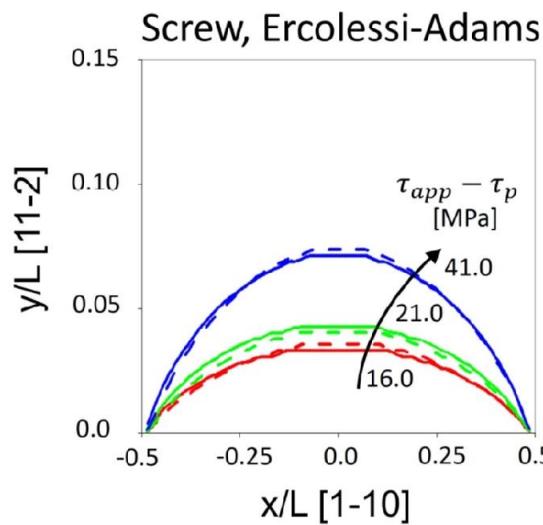


Image forces reduced on screw dislocations for this particular configurations

Edge disl. bow out approximately twice as much then screw disl., continuum model of line tension normally uses a value close to 1/4

## Summary

MD/FE in LAMMPS with a *fix*

Easy to implement and customize

Parallel with good scaling

Accurate - dislocations get to  $\approx 10\text{\AA}$  from interface with essentially zero spurious forces

Mesh generation (it is hard in 3D!) for edge and screw disl., atomic mesh resolution with this methodology

*Thank you for your attention!*